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Optocoupler-Based Power Measurement

Abstract

This note provides a tutorial on the operation of a power measurement circuit published by **W. Stephen Woodward**. This circuit makes good use of the fundamental physics behind the operation of optoisolators (phototransistors and LEDs) to construct a simple and effective means for measuring both the power and the VA consumption of an AC-powered device. This note focuses on the power consumption aspect of the circuit. A later note will deal with the VA portion of the circuit.

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Introduction

Figure 1 illustrates the circuit in question. This circuit was published in EDN magazine

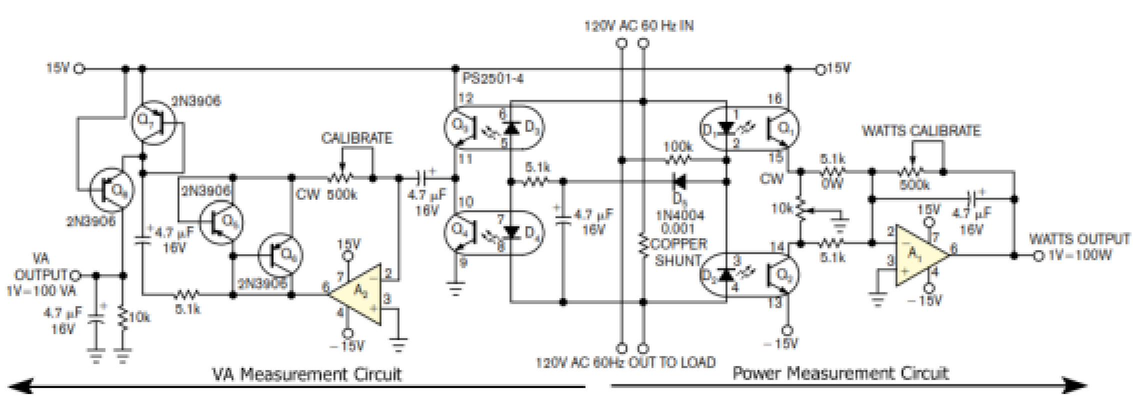


Figure 1: Circuit for Measuring Watts and VA Usage of an AC Load.

on March 3, 2008 (see **article**). The circuit produces two analog levels: (1) smoothed version of the instantaneous power usage, and (2) a smooth version of the VA usage.

This note will focus on the power measurement portion (right hand side). The VA portion is similar in operation and will be examined in a later note. Given electrical representations of power and VA, it becomes possible to compute the power factor in hardware (not addressed in this note at all).

Analysis

The circuit computes the power usage in two phases:

- Use the PN junction characteristics of two optocoupler LEDs to create two currents whose difference is proportional to power.
- Use the phototransistors of the two optocouplers to feed an opamp-based current differencing amplifier and low-pass filter that will generate a smoothed voltage proportional to the load power.

These phases are described in turn below.

LED Current Measurement

Figure 2 shows how this circuit forms the product of the load current and the AC voltage.

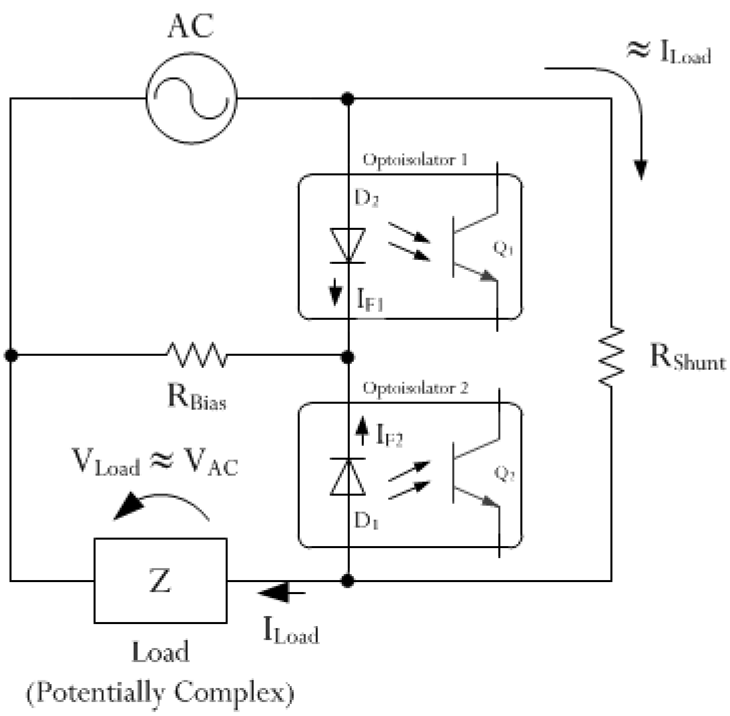


Figure 2: Circuit for Sampling Load Current.

The LEDs in this circuit each generate light proportional to the amount of current flowing through them. The current through each LED is given by the following expressions.

$$I_{D1} = I_S \cdot e^{\frac{V_{D1}}{V_T}} \qquad I_{D2} = I_S \cdot e^{\frac{V_{D2}}{V_T}} = I_S \cdot e^{\frac{V_{D1} - I_{Load} \cdot R_{Shunt}}{V_T}}$$

Where:

- I_{D1} and I_{D2} are the LED currents.
- V_{D1} and V_{D2} are the voltages across the LEDs.
- V_T is the **thermal voltage**.
- I_{Load} is the current through the load (ignoring the current through upper LEDs).
- R_{Shunt} is a resistor used to sense the current through the load.
- I_S is the **diode saturation current**.

Without loss of generality, we can base our on-going calculations on the value of I_{D1}. We can approximate I_{D1} with the following expression.

$$\therefore \quad I_{D1'} = \frac{V_{Load}}{R_{Bias}}$$

Assumes that the drop across the LED D1 is small compared to the AC voltage.

We can compute I_{D2} in terms of I_{D1} by using the diode transconductance, g_m.

$$I_{D2} = I_{D1} - g_m \cdot \delta V_D$$

where δV_D is the voltage diff. between D1 and D2.

$$I_{D2} = I_{D1} - \frac{I_{D1}}{V_T} \cdot I_{Load} \cdot R_{Shunt}$$

Assumes that $I_{Load}\, R_{Shunt} << V_D$.

As shown below, this expression allows us to express the load power in terms of the difference between the two LED currents.

$$\therefore \quad I_{D1} - I_{D2} = \frac{R_{Shunt}}{R_{Bias} \cdot V_T} \cdot I_{Load} \cdot V_{Load} = \frac{R_{Shunt}}{R_{Bias} \cdot V_T} \cdot P_{Load}$$

Expression in terms of instantaneous power.

Converting Current to Power

Once we have the two diode currents, converting them to a voltage that is proportional to power is easy. We can use a standard inverting opamp circuit to compute a voltage proportional to the difference in the currents. Consider the circuit of Figure 3.

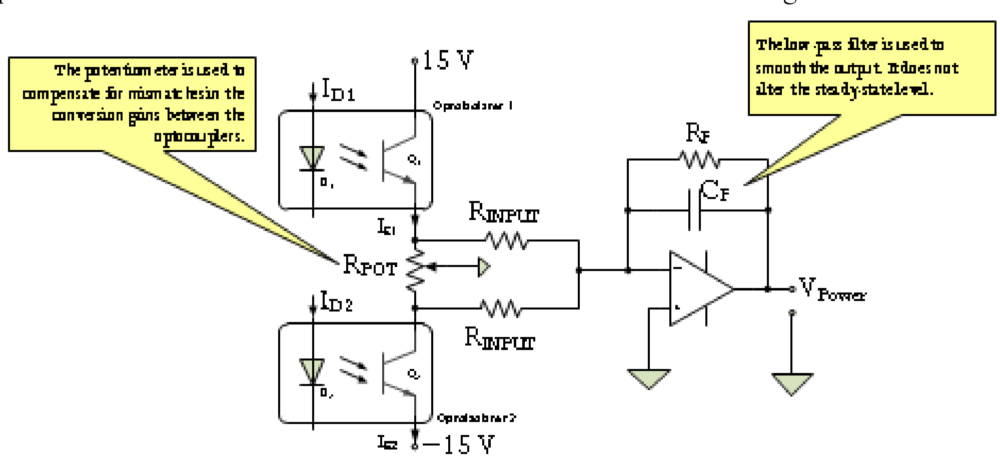


Figure 3: Circuit for Conversion of Power to Voltage Output.

To understand how this circuit works, it is useful to review the operational characteristics of the optocouplers.

The current output from a phototransistor-based optocouplers is characterized by the Current Transfer Ratio (CTR). The current transfer ratio is **defined as**:

> *A current transfer ratio (CTR) of an optocoupler indicates the rate of an output current I_C of its phototransistor to a forward input current (I_D) flowing through its light-emitting diode (LED).*

Given this definition, we can define the optocoupler CTR using the following equation.

$$CTR \equiv \frac{I_C}{I_D}$$

Where

- I_C is the collector current of the optotransistor
- I_D is the input LED current.

In general, CTR is not a constant. Figure 4 illustrates the CTR of a typical optocoupler as a function of the LED current.

Source

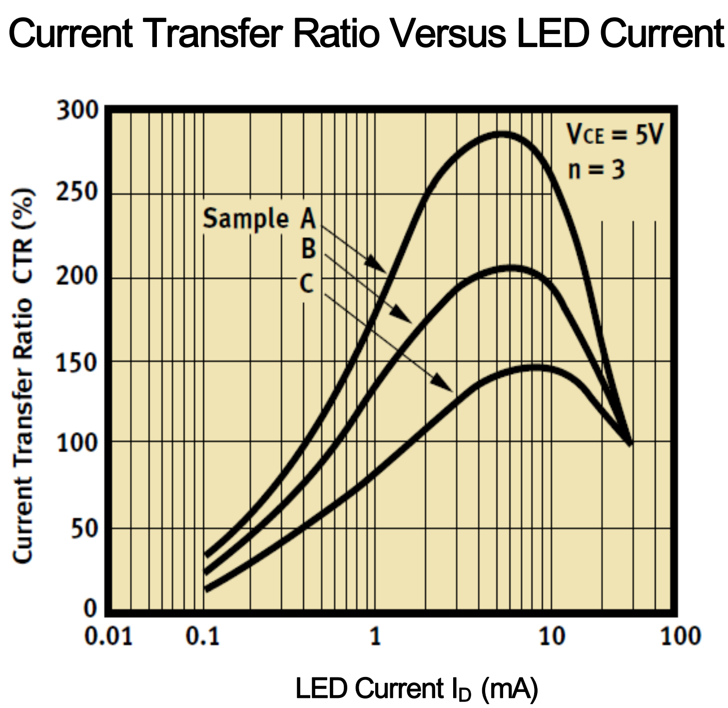


Figure 4: Current Transfer Ratio for a Typical.

Optocoupler.

This circuit assumes we are running the phototransistors at current levels that are sufficiently linear to provide the accuracy that we required. Under this assumption, we can model the output currents from the two optocouplers using the following equations.

$$I_{C1} = CTR_1 \cdot I_{D1} \qquad I_{C2} = CTR_2 \cdot I_{D2}$$

Where:

- I_{C1} is the collector current in phototransistor Q1.
- I_{C2} is the collector current in phototransistor Q2.
- CTR_1 is the current transfer ratio for optocoupler 1.
- CTR_2 is the current transfer ratio for optocoupler 2.

The optocoupler CTRs can vary even within the same the part. By varying the R_{POT} wiper setting, the R_{POT} resistance can be used to form two current dividers that will ensure the effective CTRs (i.e. CTR after the potentiometer-based current divider) from the two optocouplers are equalized, at least at one operating point.

Lets define the variable CTR to be equal the effective CTRs of optocouplers after the R_{POT} wiper has been adjusted to equalize them.

$$V_{Power} = \frac{R_F}{R_{Input}} \cdot \left(I_{C1} - I_{C2}\right)$$

$$V_{Power} = \frac{R_F}{R_{Input}} \cdot CTR \cdot \left(I_{D1} - I_{D2}\right)$$

$$\therefore \quad V_{Power} = CTR \frac{R_F}{R_{Input}} \cdot \frac{R_{Shunt}}{R_{Bias} \cdot V_T} \cdot P_{Load}$$

The load power is proportional to the output voltage of the opamp.

Note that I have not actually proven that the potentiometer can be used to adjust the effective CTRs of both optocouplers. This can be easily done but the derivation is a bit busy. Thus, I have included the details in the **Appendix** of this document.

Conclusion

I have verified that the Woodward circuit will generate a voltage proportional to the load power. The circuit does require a calibration operation with a potentiometer.

There are assumptions that were made in the design of the circuit that are worth reviewing here.

- R_{Shunt} is small relative to the impedance in the load.
- The CTR of the optocouplers can be modeled as a constant (they do not have to have the same constant)

- The LED currents are small enough as to be ignorable with respect to the load current.
- The LED voltages are sufficiently close that we can model the transconductance as a constant.

Appendix

Demonstration of CTR Equalization Using A Potentiometer

We have not shown that a single potentiometer can be used to force the effective CTRs of the optocouplers to be equal. Assume that potentiometer resistance is divided between the ratios by the ratio k, which means that the upper optocoupler is assigned $k \cdot R_{POT}$ and the lower optocoupler is assigned $(1\text{-}k) \cdot R_{POT}$.

$$Effective\_CTR_1 = Effective\_CTR_2$$

$$CTR_1 \cdot \frac{k \cdot R_{POT}}{k \cdot R_{POT} + R_{Input}} = CTR_2 \cdot \frac{(1-k) \cdot R_{POT}}{(1-k) \cdot R_{POT} + R_{Input}}$$

$$eq1 := CTR_1 \cdot \frac{k \cdot R_{POT}}{k \cdot R_{POT} + R_{Input}} = CTR_2 \cdot \frac{(1-k) \cdot R_{POT}}{(1-k) \cdot R_{POT} + R_{Input}} \;\Bigg|\; \begin{array}{l} \text{substitute}, k = \frac{1}{2} + \varepsilon \\ \text{substitute}, R_{POT} = 2 \cdot R_{Input} \end{array} \rightarrow$$

$$eq1 := eq1 \text{ solve}, \varepsilon \rightarrow \begin{pmatrix} \dfrac{CTR_1 + CTR_2 + \sqrt{9 \cdot CTR_1^{\,2} - 14 \cdot CTR_1 \cdot CTR_2 + 9 \cdot CTR_2^{\,2}}}{4 \cdot CTR_1 - 4 \cdot CTR_2} \\ \dfrac{CTR_1 + CTR_2 - \sqrt{9 \cdot CTR_1^{\,2} - 14 \cdot CTR_1 \cdot CTR_2 + 9 \cdot CTR_2^{\,2}}}{4 \cdot CTR_1 - 4 \cdot CTR_2} \end{pmatrix}$$

$$F(CTR_1, CTR_2) := eq1_1 \rightarrow \frac{CTR_1 + CTR_2 - \sqrt{9 \cdot CTR_1^{\,2} - 14 \cdot CTR_1 \cdot CTR_2 + 9 \cdot CTR_2^{\,2}}}{4 \cdot CTR_1 - 4 \cdot CTR_2}$$

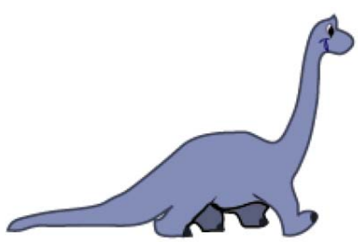
Example

$CTR_1 := 10\%$ ***Assume badly mismatched optocouplers***

$CTR_2 := 90\%$

$\varepsilon := F(CTR_1, CTR_2) = 0.461$

$k := \frac{1}{2} + \varepsilon = 0.961$ $R_{POT} := 10000$ $R_{Input} := 5000$



$$effectiveCTR_1 := CTR_1 \cdot \frac{k \cdot R_{POT}}{k \cdot R_{POT} + R_{Input}} = 0.066$$

$$effectiveCTR_2 := CTR_2 \cdot \frac{(1-k) \cdot R_{POT}}{(1-k) \cdot R_{POT} + R_{Input}} = 0.066$$