Solution

Random Index Table

RI := (0 0 0.58 0.89 1.12 1.24 1.32 1.41 1.45) **Defined in the AHP algorithm**

Utility Function

$$n(x) := \left| \begin{array}{l} i \leftarrow 0 .. \ rows(x) - 1 \\ \dfrac{x}{\left(\sum_i x_i \right)} \end{array} \right.$$

Raw Performance Figures

Emissions :=

	0
0	110
1	104
2	0

Grams of CO2 per km

Range :=

	0
0	448
1	896
2	100

Miles per Tankful

FuelCost :=

	0
0	12.6
1	13.2
2	3.1

$ per mile

VehicleCost :=

	0
0	$2.51 \cdot 10^4$
1	$2.636 \cdot 10^4$
2	$3.953 \cdot 10^4$

$ per car

Generate Comparable Performance Indices

For a performance index, higher is always better and zero is not good to use because it does not allow ratios relative to it. Our scale is from 1 (reference) to 10 (extremely good). I will use a linear model. Nonlinear models are possible, but not used for this exercise.

Emissions

I normalized the grading

intercept
slope
$$\begin{pmatrix} i \\ m \end{pmatrix} := \text{line}\left[\begin{pmatrix} 110 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 10 \end{pmatrix}\right] = \begin{pmatrix} 10 \\ -0.082 \end{pmatrix} \Rightarrow \text{grade}_E := (m \cdot \text{Emissions} + i)\ n = \begin{pmatrix} 0.08 \\ 0.119 \\ 0.801 \end{pmatrix}$$

Fuel Costs

intercept
slope
$$\begin{pmatrix} i \\ m \end{pmatrix} := \text{line}\left[\begin{pmatrix} 13.2 \\ 3.1 \end{pmatrix}, \begin{pmatrix} 1 \\ 10 \end{pmatrix}\right] = \begin{pmatrix} 12.762 \\ -0.891 \end{pmatrix} \Rightarrow \text{grade}_{FC} := (m \cdot \text{FuelCost} + i)\ n = \begin{pmatrix} 0.122 \\ 0.08 \\ 0.798 \end{pmatrix}$$

range

intercept
slope
$$\begin{pmatrix} i \\ m \end{pmatrix} := \text{line}\left[\begin{pmatrix} 100 \\ 899 \end{pmatrix}, \begin{pmatrix} 1 \\ 10 \end{pmatrix}\right] = \begin{pmatrix} -0.126 \\ 0.011 \end{pmatrix} \Rightarrow \text{grade}_R := (m \cdot \text{Range} + i)\ n = \begin{pmatrix} 0.31 \\ 0.627 \\ 0.063 \end{pmatrix}$$

Vehicle Cost

intercept
slope
$$\begin{pmatrix} i \\ m \end{pmatrix} := \text{line}\left[\begin{pmatrix} 22000 \\ 40000 \end{pmatrix}, \begin{pmatrix} 10 \\ 1 \end{pmatrix}\right] = \begin{pmatrix} 21 \\ -5 \times 10^{-4} \end{pmatrix} \Rightarrow \text{grade}_{VC} := (m \cdot \text{VehicleCost} + i)\ n = \begin{pmatrix} 0.483 \\ 0.447 \\ 0.07 \end{pmatrix}$$

Criteria Pairwise Comparison

I will use the reference comparisons done as part of the NASA example.

	E	FC	R	VC	
$PC :=$	1	1	5	5	Emissions
	1	1	5	5	Fuel Cost
	$\frac{1}{5}$	$\frac{1}{5}$	1	3	Range
	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{3}$	1	Vehicle Cost

$$ePC := eigenvals(PC) = \begin{pmatrix} 4.155 \\ -0.077 + 0.797i \\ -0.077 - 0.797i \\ 0 \end{pmatrix}$$

$$\lambda_{max} := Re(max(ePC)) = 4.155$$

$$\mu := \frac{\lambda_{max} - rows(PC)}{rows(PC) - 1} = 0.052$$

$$I := \frac{\mu}{RI_{0,(rows(PC)-1)}} = 0.058$$

The preference vector will be used to weight the important of the four criteria

$$Preference := \left| \begin{array}{l} i \leftarrow 0..\ rows(PC) - 1 \\ eigenvec(PC, \lambda_{max})\ n \end{array} \right. = \begin{pmatrix} 0.41 \\ 0.41 \\ 0.11 \\ 0.06 \end{pmatrix}$$

Emissions

Fuel Cost

Range

Vehicle Cost

Scoring

$$Criteria := augment(grade_E, grade_{FC}, grade_R, grade_{VC}) = \begin{pmatrix} 0.08 & 0.122 & 0.31 & 0.483 \\ 0.119 & 0.08 & 0.627 & 0.447 \\ 0.801 & 0.798 & 0.063 & 0.07 \end{pmatrix}$$

I put the scoring of the alternatives into a single array.

$$Preference = \begin{pmatrix} 0.411 \\ 0.411 \\ 0.113 \\ 0.064 \end{pmatrix}$$

Emissions
Fuel Cost
Range
Vehicle Cost

$$\alpha := Criteria \cdot Preference = \begin{pmatrix} 0.149 \\ 0.182 \\ 0.669 \end{pmatrix}$$

Propane
Hybrid
Electric

This is the final scoring results

Final Results

PAIR-WISE COMPARE	DECISION CRITERIA			
	Emissions	Fuel Cost	Range	Vehicle Cost
Emissions	1.00	1.00	5.00	5.00
Fuel Cost	1.00	1.00	5.00	5.00
Range	0.20	0.20	1.00	3.00
Vehicle Cost	0.20	0.20	0.33	1.00

CRITERIA MATRIX	Emissions	Fuel Cost	Range	Vehicle Cost
Propane	0.08	0.12	0.31	0.48
Hybrid Electric	0.12	0.08	0.63	0.45
Electric	0.80	0.80	0.06	0.07

x

Pref. Vector	
0.41	Emissions
0.41	Fuel Cost
0.11	Range
0.06	Vehicle Cost

	GREEN VEHICLE OPTIONS		
MATRIX CALCULATIONS	Propane	Hybrid Elec.	Electric
Final Scores	0.15	0.18	0.67
SELECTED VEHICLE			✓

$(PC \quad Criteria \quad \alpha^T \quad Preference)$