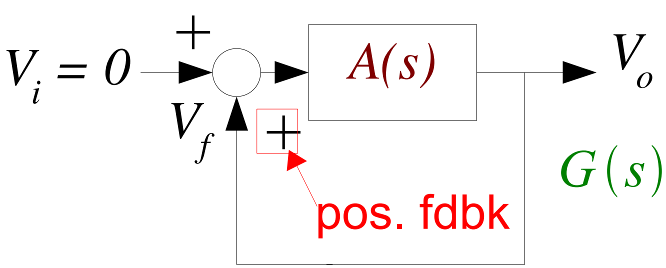
Colpitts Oscillator

- Basic positive feedback oscillator
- The Colpitts LC Oscillator circuit
- Open-loop analysis
- Closed-loop analysis
- Root locus
- Stability limit
- Colpitts design

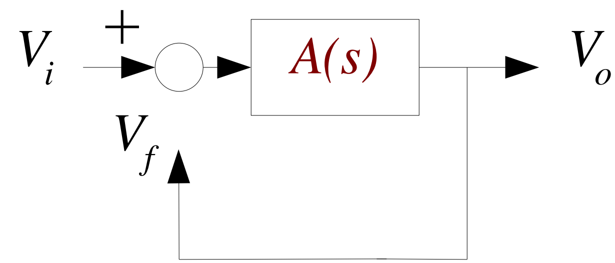
Basic Positive Feedback Oscillator

closed-loop oscillator



$$G(s)=\frac{A(s)}{1-A(s)}$$

open-loop: determine loop-gain



$$V_o=A(s)(V_i+V_f)=A(s)(0+V_o)\Rightarrow$$

$$V_o(1-A(s))=0$$

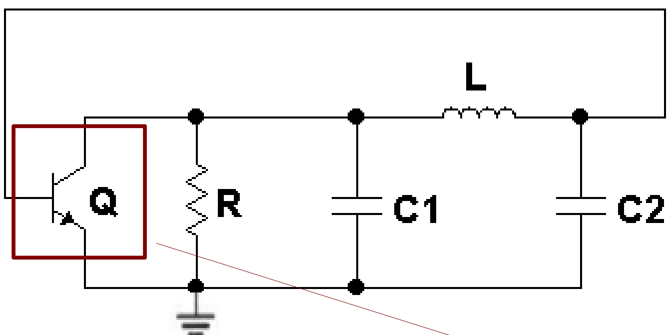
$$\frac{V_f}{V_i}=\frac{V_o}{V_i}=A(s)=\frac{K\,N(s)}{D(s)}$$

Since: $V_o\neq 0\Rightarrow 1-A(s)=0\Rightarrow A(s)=1\Rightarrow \boxed{D(s)-K\,N(s)=0}$

Condition for oscillation at $s=j\omega_0$: $\boxed{A(s)=1\,e^{j\pm 2k\pi}}$ Barkhausen criterion

for $k = 0, 1, 2$ ..

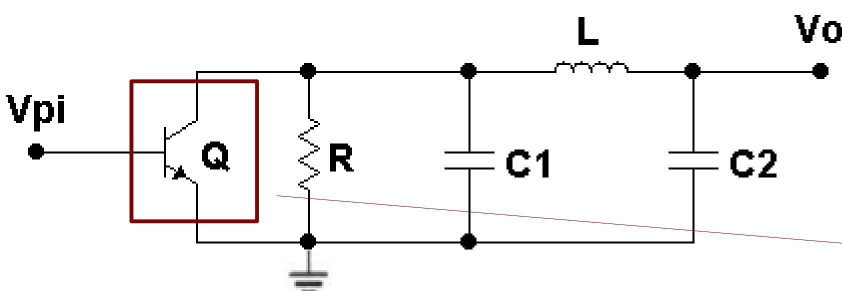
Colpitts Oscillator Basic Schematic



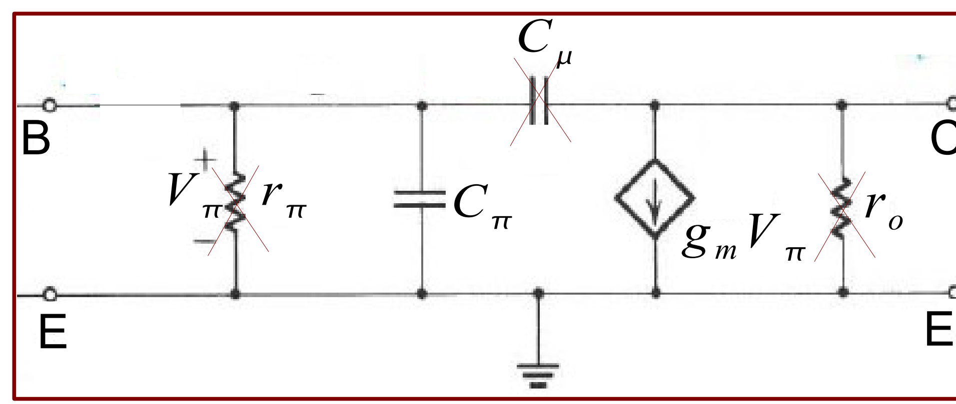
Emphasizing feedback

Assumptions:

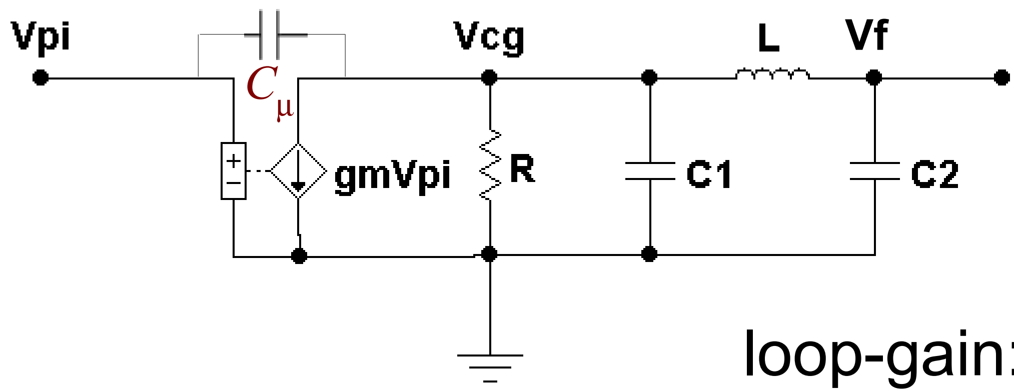
1. r_π large (compared to $1/\omega C_2$).
2. C_μ negligible (compared to C_1, C_2)
3. C_π part of C_2 *(in closed loop)*
4. R represents total resistance in collector circuit, i.e. $R \| r_o \approx R$



Emphasizing loop gain $\Gamma_F A(s)$



Loop-Gain Analysis



loop-gain: $A(s)=\dfrac{V_f}{V_\pi}$

Node equation at v_{cg}:

$C_\mu \ll C_1$

$$(g_m - sC_\mu)V_\pi + \frac{V_{cg}}{R} + s(C_1 + C_\mu)V_{cg} + \frac{(V_{cg}-V_f)}{sL} = 0$$

at v_f:

$$\frac{(V_f - V_{cg})}{sL} + sC_2V_f = 0$$

Note that

$V_f = V_f(s)$

$V_{cg} = V_{cg}(s)$

$V_\pi = V_\pi(s)$

Open Loop Analysis - cont.

from previous slide

$$C_\mu \ll C_1$$

$$(g_m - sC_\mu)V_\pi + \frac{V_{cg}}{R} + s(C_1 + \cancel{C_\mu})V_{cg} + \frac{(V_{cg} - V_f)}{sL} = 0$$

$$\frac{(V_f - V_{cg})}{sL} + sC_2 V_f = 0$$

Rearranging the two equations:

$$\left(\boxed{sC_1 + \frac{1}{sL}} + \frac{1}{R}\right)V_{cg} - \frac{1}{sL}V_f = (sC_\mu - g_m)V_\pi$$

$$-\frac{1}{sL}V_{cg} + \left(\boxed{sC_2 + \frac{1}{sL}}\right)V_f = 0$$

Further rearrangement:

$$\left(\boxed{\frac{s^2 LC_1 + 1}{sL}} + \frac{1}{R}\right)V_{cg} - \frac{1}{sL}V_f = (sC_\mu - g_m)V_\pi$$

$$-\frac{1}{sL}V_{cg} + \left(\boxed{\frac{s^2 LC_2 + 1}{sL}}\right)V_f = 0$$

Open Loop Analysis - cont.

from previous slide

$$\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_{cg}-\frac{1}{sL}V_f=(sC_\mu-g_m)V_\pi \quad (1)$$

$$-\frac{1}{sL}V_{cg}+\left(\frac{s^2LC_2+1}{sL}\right)V_f=0 \quad (2)$$

Prepare to add the two equations:

$$\frac{1}{sL}\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_{cg}-\left(\frac{1}{sL}\right)^2V_f=\frac{sC_\mu-g_m}{sL}V_\pi$$

$$Eq(1)*\frac{1}{sL}$$

$$-\frac{1}{sL}\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_{cg}+\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)\left(\frac{s^2LC_2+1}{sL}\right)V_f=0$$

$$Eq(2)*\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)$$

Adding (V_{cg} terms cancel):

$$\left(-\left(\frac{1}{sL}\right)^2+\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)\left(\frac{s^2LC_2+1}{sL}\right)\right)V_f=\frac{sC_\mu-g_m}{sL}V_\pi$$

Open Loop Analysis – cont.

From previous slide

$$\left(-\left(\frac{1}{sL}\right)^2+\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)\left(\frac{s^2LC_2+1}{sL}\right)\right)V_f=\frac{C_\mu-g_m}{sL}V_\pi$$

Multiply by $(sL)^2$:

$$\left(-1+\left(s^2LC_1+1+\frac{sL}{R}\right)\left(s^2LC_2+1\right)\right)V_f=(sC_\mu-g_m)sLV_\pi$$

Expand and collect terms according to s^n:

$$\left(-1+s^4C_1C_2L^2+s^2\left(LC_1+LC_2\right)+s^3\frac{L^2C_2}{R}+s\frac{L}{R}+1\right)V_f=(sC_\mu-g_m)sLV_\pi$$

Open Loop Analysis - cont.

From previous slide

$$\left(\cancel{-1}+s^4 C_1 C_2 L^2+s^2\left(L C_1+L C_2\right)+\frac{s^3 L^2 C_2}{R}+\frac{s L}{R}+\cancel{1}\right)V_f=(s C_\mu-g_m)\, sLV_\pi$$

Canceling (-1 by 1) and dividing by sL:

$$\left(s^3 C_1 C_2 L+s\left(C_1+C_2\right)+\frac{s^2 L C_2}{R}+\frac{1}{R}\right)V_f=(s C_\mu-g_m)V_\pi$$

Multiply by R:

$$\left(s^3 R C_1 C_2 L+s R\left(C_1+C_2\right)+s^2 L C_2+1\right)V_f=(s C_\mu-g_m)R V_\pi$$

Open Loop Analysis - cont.

From previous slide

$$\left(s^3 R C_1 C_2 L + s R\left(C_1+C_2\right) + s^2 L C_2 + 1\right) V_f = (s C_\mu - g_m) R V_\pi$$

Normalize (divide by RC_1C_2L) and factor out C_μ:

$$\left(s^3 + s\frac{\left(C_1+C_2\right)}{C_1 C_2 L} + s^2\frac{1}{R C_1} + \frac{1}{R C_1 C_2 L}\right) V_f = \frac{(s - \frac{g_m}{C_\mu}) R C_\mu}{R C_1 C_2 L} V_\pi$$

The loop-gain transfer function:

$$\frac{V_f}{V_\pi} = A(s) = \frac{\frac{R C_\mu}{R C_1 C_2 L}(s - \frac{g_m}{C_\mu})}{s^3 + s^2\frac{1}{R C_1} + s\frac{\left(C_1+C_2\right)}{C_1 C_2 L} + \frac{1}{R C_1 C_2 L}} = \frac{K N(s)}{D(s)}$$

Closed Loop Analysis - cont.

The closed loop equation (note the positive feedback):

$$T(s)=\frac{A(s)}{1\boxed{-A(s)}}=\frac{KN(s)}{D(s)\boxed{-KN(s)}}$$

$$A(s)=\frac{\frac{RC_\mu}{RC_1C_2L}\left(s-\frac{g_m}{C_\mu}\right)}{s^3+s^2\frac{1}{RC_1}+s\frac{(C_1+C_2)}{C_1C_2L}+\frac{1}{RC_1C_2L}}=\frac{KN(s)}{D(s)}$$

where:

$$A(s)=\frac{KN(s)}{D(s)} \qquad K=\frac{RC_\mu}{RC_1C_2L} \qquad N(s)=s-\frac{g_m}{C_\mu}\approx-\frac{g_m}{C_\mu}$$

$$D(s)=s^3+s^2\frac{1}{RC_1}+s\frac{(C_1+C_2)}{C_1C_2L}+\frac{1}{RC_1C_2L}$$

then:

$$D(s)-KN(s)=D(s)-\frac{-g_mR}{RC_1C_2L}=s^3+s^2\frac{1}{RC_1}+s\frac{(C_1+C_2)}{C_1C_2L}+\frac{1+g_mR}{RC_1C_2L}$$

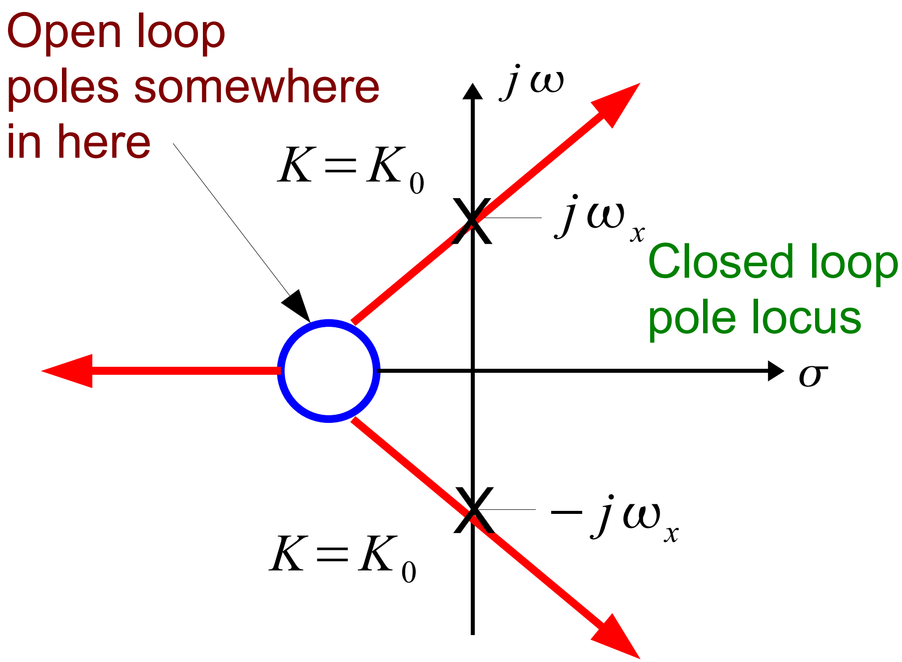
Closed Loop Analysis - cont.

We know that the open loop system $A(s)$ is stable. It has poles in the left-half s-plane, since it is a passive RLC circuit. We also know that it has 3 stable poles. One is negative-real, the other 2 can be negative-real or LHP complex conjugates.

$$D(s)=s^3+s^2\frac{1}{RC_1}+s\frac{(C_1+C_2)}{C_1C_2L}+\frac{1}{RC_1C_2L}$$

So, let's do a rough sketch of the root locus for a feedback system with a 3 stable pole $A(s)$.

Root Locus Characteristic



The loop will become unstable for any value of $K>K_0$.

Rather than sketch the root locus in more exacting detail – it has served its purpose by verifying that oscillation is possible.

Let's solve for the required K_0 .

Stability Limit Calculation

If the closed-loop system is at the stability limit point:

$$D(s)=(s+a)(s^2+\omega_x^2)$$

to oscillate

Multiplying terms:

$$D(s)=s^3+a\,s^2+\omega_x^2 s+a\,\omega_x^2 \Rightarrow D(j\omega)=\overbrace{(a\,\omega_x^2-a\,\omega^2)}^{=0}=j\overbrace{(\omega_x\omega-\omega^3)}^{=0}$$

Match term by term with:

$$D(s)+K\,N(s)=s^3+s^2\overset{a}{\boxed{\frac{1}{RC_1}}}+s\overset{\omega_x^2}{\boxed{\frac{(C_1+C_2)}{C_1C_2L}}}+\overset{a\,\omega_x^2}{\boxed{\frac{1+g_mR}{RC_1C_2L}}}$$

$$a=\frac{1}{RC_1}\qquad \omega_x^2=\frac{1}{\left(\dfrac{C_1C_2}{C_1+C_2}\right)L}\qquad a\,\omega_x^2=\frac{1+g_mR}{RC_1C_2L}$$

Stability Limit

$$a=\frac{1}{RC_1}$$

$$\boxed{\omega_x^2=\frac{1}{\left(\frac{C_1C_2}{C_1+C_2}\right)L}=\frac{C_1+C_2}{C_1C_2L}}$$

$$a\,\omega_x^2=\frac{1+g_mR}{RC_1C_2L}$$

To find the “gain” requirement for oscillation, equate:

$$\underbrace{\frac{1}{RC_1}}_{a}\underbrace{\frac{C_1+C_2}{C_1C_2L}}_{\omega_x^2}=\frac{C_1+C_2}{RC_1^2C_2L}=\underbrace{\frac{1+g_mR}{RC_1C_2L}}_{a\,\omega_x^2}\Rightarrow 1+g_mR=\frac{C_1+C_2}{C_1}$$

$$\boxed{g_mR=\frac{C_1+C_2-C_1}{C_1}=\frac{C_2}{C_1}}$$

Oscillator Design Summary

The oscillation frequency:

$$\boxed{\omega_x=\sqrt{\frac{C_1+C_2}{C_1 C_2 L}}}$$

The required feedback gain C_2/C_1:

$$g_m R=\frac{C_2}{C_1}$$

To insure the start-up of oscillation:

$$\boxed{g_m R>\frac{C_2}{C_1}}$$

Comments:

1. Unfortunately we don't control "R".
2. We can fix g_m and adjust C_2/C_1 and adjust "L" to keep ω_x constant.
2. We can adjust g_m through the bias current I_C and set C_2 and C_1 at convenient values, say $C_2 = C_1 = C$. We can know choose L.

Practical Colpitts Oscillator Circuit

$RFC = RF\ choke \rightarrow$ large reactance at ω_0 / low resistance at dc