University *of Ljubljana*
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NEAR-EARTH OBJECTS

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Abstract

This seminar is talking about small bodies in Solar system, which have trajectories in the vicinity of Earth's orbit around the Sun. These so-called Near-Earth Objects (NEOs) are passing Earth closer than any other planetary bodies and sometimes collide with Earth. To minimize the collision threat, search programs for NEOs are operating worldwide and trying to complete the catalog of NEOs. Astrometric and photometric measurements enable orbit calculations and also an estimations about NEO's diameter, mass, collision probability and expected impact energy.

Contents

1 Introduction

A near-Earth object (NEO) is a Solar System object whose orbit brings it into close proximity with the Earth [1]. Every day, the Earth is bombarded with more than 100 tons of dust and sand-size particles – meteoroids. Many of the incoming particles are so small that they are destroyed and completely vaporized in the Earth's atmosphere before they reach the ground. These particles are often seen as meteors (shooting stars). The vast majority of all interplanetary material that reaches the Earth's surface originates as the collision fragments of asteroids that have run into one another. The collisions of asteroids and comets with the Earth are less frequent, but very important for having significantly modified the Earth's biosphere in the past and will continue to do so in the future. With an average interval of about 100 years, rocky or iron asteroids larger than about 50 meters would be expected to reach the Earth's surface and cause local disasters or produce the tidal waves that can inundate low lying coastal areas. On an average of every few hundred thousand years or so, asteroids larger than 1 km could cause global disasters. The probability of an asteroid striking the Earth and causing serious damage is very remote but the devastating consequences of such an impact suggests we should closely study different types of asteroids to understand their compositions, structures, sizes, and future trajectories.

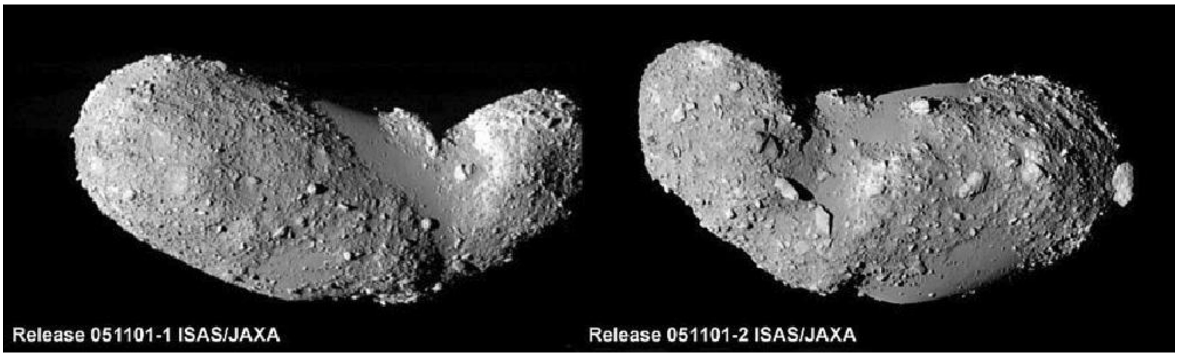

Figure 1: Near-Earth asteroid 25143 Itokawa (dimensions 540 meters by 270 meters by 210 meters) observed by Hayabusa [2]. Itokawa was the first asteroid to be the target of a sample return mission - the Japanese space probe Hayabusa. Hayabusa arrived in the vicinity of Itokawa on 12th September 2005 and initially "park" in an asteroid-Sun line at 20 km and later 7 km from the asteroid. After arriving at Itokawa, Hayabusa studied the asteroid's shape, spin, topography, colour, composition, density, and history. On 20th November 2005 Hayabusa landed for thirty minutes, but failed to operate a device designed to collect soil samples. On November 25th, a second landing and sampling sequence was attempted. From then Hayabusa has left the asteroid and the sample capsule is planned to land at Woomera (South Australia) in 2010. However, it is unclear if any samples were collected.

2 Terminology

2.1 Planet, Dwarf Planet or Minor Planet?

In the past there was some confusion around the term "**asteroid**" which comes from Greek and means star-like, star-shaped. It was proposed by the astronomer Sir William Herschel in 1802, because these new objects were star-like in that they were unresolvable points of light in a telescope. However, this term was not universally adopted at that time. By the mid 1800s, after several dozen of this bodies had been discovered, the French and Germans referred to them as "small" planets, while the British Royal Astronomical Society officially called them **minor planets**. Until modern times, the term "asteroid" was mostly used by astronomers in America and has historically been applied primarily to minor planets of the inner Solar System, as the outer Solar System was poorly known when it came into common usage.

The discovery of hundreds of **trans-Naptunian objects**[1] (**TNOs**), one of them larger than Pluto, caused astronomers to ask both the specific questions, Is Pluto a planet? and the more general question, What is a planet? In 2006, the International Astronomical Union (IAU) approved a definition of the word "planet" that seemed, to the majority of astronomers who voted on the issue, the most useful way of defining that word [3]. According to the IAU definition, an object within the solar system is a planet if

1. It is in orbit around the Sun, and is not a satellite of another planet.
2. It has sufficient mass for its self-gravity to overcome its compressional strength, and thus assume a spherical, or spheroidal, shape in hydrostatic equilibrium.
3. It has cleared its orbital neighborhood (this criterion is sometimes referred to as "orbital dominance").

The four terrestrial planets and four Jovian planets satisfy all these criteria, and so they are labeled "planets"[2]. Objects that satisfy the first two criteria, but not the last, are called "**dwarf planets**". Till now IAU confirmed 5 dwarf planets (table 1), though dozens of others are thought likely to be so classified in the future. In 2008 there were more than 70 candidates for dwarf planets in the Kuiper belt and they are estimating that there are another ~ 2000 dwarf planets beyond Kuiper belt in the region where Sedna resides (Oort cloud).

2.2 Small Solar System Bodies

If an object orbits around the Sun and is not a planet or a dwarf planet nor a satellite of another body, then it is a **Small Solar System Body** (**SSSB**) according to resolution of IAU [3]. The 2006 definition of SSSB says that they "*include most of the Solar System asteroids, most Trans-Neptunian Objects (TNOs), comets, and other small bodies*". SSSB

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[1] Trans-Naptunian objects are icy and rocky bodies found primarily in the Kuiper belt beyond the orbit of Neptune ($a > 30$ AU). Kuiper belt is the region close to the ecliptic plane at distances from $\sim 30\,AU$ to $\sim 50\,AU$ from the Sun.

[2] Terrestrial (Earth-like) planets or inner planets are Mercury, Venus, Earth and Mars. Jovian (Jupiter-like) planets are Jupiter, Saturn, Uranus and Neptune.

Table 1: At the moment confirmed dwarf planets, 5 biggest asteroids and 4 biggest dwarf planets candidates [4].

Name	**Diameter (km)**	**Mass ($M_\otimes$)**[a]	**Semimajor Axis (AU)**
Dwarf Planets			
Eris	2500	1.00	67.90
Pluto	2400	0.79	39.45
Makemake	1500	∼0.25	45.43
Haumea	1400	0.25	43.13
Ceres	940	0.057	2.767
Minor Planets (Asteroids)			
Vesta	576	0.016	2.362
Pallas	538	0.013	2.771
Hygeia	430	0.005	3.144
Interamnia	338	0.002	3.062
Davida	324	0.003	3.178
Dwarf Planets Candidate			
Sedna	<1600	0.1-0.4	510
2007$OR_1$0	900-1400	unknown	67
Orcus	875-1020	0.04	39
Quaoar	655-1050	0.06-0.16	44

[a] $M_\otimes = 1.67 \times 10^{22}\, kg$ (This is the mass of Eris, the biggest known dwarf planet. Comparable to Earth it has 0.00280 of Earth mass.)

was introduced to cover both minor plantes and comets, because the distinction between them is also based on our observational qualities rather then any inherent difference in physical properties or composition. **Comets** are characterized by their coma, or cloud of sublimating gas and expelled dust. This gives them their characteristic diffuse fuzzy halo and long streaming tail. But comets only become "cometary" when they enter the inner solar system and are heated sufficiently by the Sun to evaporate their volatile materials. The point at which frozen volatiles begin to sublimate can vary depending on composition, but for most comets this is approximately at 4 AU[3]. A number of outer solar system objects that could be called minor planets (or asteroids) may be composed of the same collection of volatile ices, dust, metal and carbonaceous organics as comets. Because their orbits are less elliptical than currently active comets, they never travel close enough to the Sun to warm their surfaces, cause their ices for flash to gas and appear cometary. These objects are solid bodies only because their surfaces stay cold enough to keep their gases frozen.

In this seminar I will be interested in the SSSBs which have orbits in the vicinity of Earth's orbit. This SSSBs are relatively close to the Sun and it is not hard to distinguish between minor planets (asteroids) and comets. So I will use the terms comets and asteroids instead of SSSBs. There are also a lot of very small asteroids which are traditionally called **meteoroids**. The root word meteor comes from the Greek *meteoros*, meaning "a thing up high". The

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[3] AU means astronomical unit and it is the mean distance between Earth and the Sun ($1.496 \times 10^{11}\, m$)

boundary between small asteroids and large meteoroids is a fuzzy one. The current official definition of a meteoroid from the IAU is "*a solid object moving in interplanetary space, of a size considerably smaller than an asteroid and considerably larger than an atom*". The British Royal Astronomical Society has proposed a new definition where a meteoroid is between $100\,\mu m$ and $10\,m$ across. The NEO definition includes larger objects, up to $50\,m$ in diameter, in this category. Once a meteoroid enters the Earth's atmosphere and vaporizes, it becomes a **meteor** (a shooting star). If a small asteroid or large meteoroid survives its fiery passage through the Earth's atmosphere and lands upon the Earth's surface, it is then called a **meteorite**. Cometary debris is the source of most small meteoroid particles. Many comets generate meteoroid streams when their icy cometary nuclei pass near the Sun and release the dust particles that were once embedded in the cometary ices. These meteoroid particles then follow in the wake of the parent comet. Collisions between asteroids in space create smaller asteroidal fragments and these fragments are the sources of most meteorites that have struck the Earth's surface.

2.3 Near-Earth Objects

Any object, such as a meteoroid, an asteroid or a comet, orbiting the Sun with the perihelion distance $q < 1.3\,AU$ and aphelion distance $Q > 0.983\,AU$ is defined as a **near-Earth object** (**NEO**). This means that this object has a part of their orbit in the vicinity of Earth's orbit (near 1.0 AU from the Sun) since the vast majority of NEOs are orbiting near ecliptic plane around the Sun. As we know a perihelion is the point in the elliptical orbit where the object is closest to the Sun. Similar the point where the object is furthest from the Sun is called "aphelion" (Figure 2). Earth's orbit is almost a perfect circle, so the difference between its distance to the Sun at aphelion and at perihelion is slight. At perihelion, our planet is about 147 million km ($q = 0.983\,AU$) from the Sun and it moves outward to around 152 million

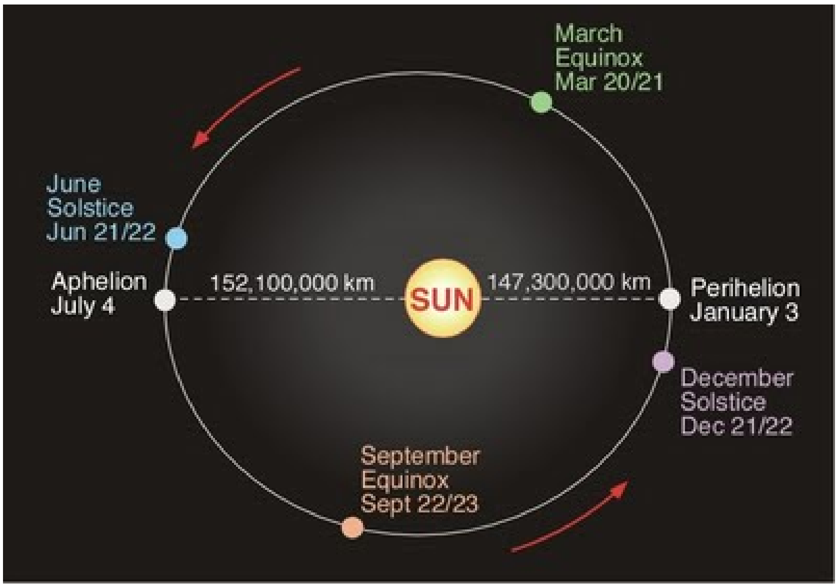


Figure 2: Earth reaches perihelion in early January each year, and passes through its aphelion point near the start of July.

km ($Q = 1.017\,AU$) from the Sun at aphelion. Earth is about 3% further from the Sun at aphelion than it is at perihelion. Some planets have orbits that are more elongated, their orbits have a greater "eccentricity", which is a technical term for how "stretched out" an orbit is. For exemple Mercury has the most eccentric orbits of the planets and it is 52% further from the Sun at aphelion than it is at perihelion (tabel 2). For elliptic orbits eccentricity (ϵ) may take values between 0 and 1. It can be calculated from distance at aphelion (Q) and perihelion (q):

$$\epsilon = \frac{Q - q}{Q + q}$$

In other respects the eccentricity of an ellipse is the ratio of the distance between the foci to the length of the major axis. Considering this it follows:

$$q = a(1 - \epsilon) \qquad Q = a(1 + \epsilon)$$

where a is the length of the semimajor axis. As determined by Kepler and stated in his Second Law of Planetary Motion, the speed of an object in its orbit is fastest at perihelion and slowest at aphelion:

$$v_{pe} = \sqrt{\frac{GM(1+\epsilon)}{a(1-\epsilon)}} \qquad v_{ap} = \sqrt{\frac{GM(1-\epsilon)}{a(1+\epsilon)}}$$

where G is the gravitational constant ($6.67300 \times 10^{-11} m^3 kg^{-1} s^{-2}$) and M is the mass of the Sun ($1.9891 \times 10^{30}\,kg$). The eccentricity, semimajor axis, speed at aphelion and speed at perihelion of the terrestrial planets are assembled in table 2.

Table 2: Some orbital elements for terrestrial planets and Jupiter [1].

Planet	$a(AU)$	ϵ	$v_{ap}(km/s)$	$v_{pe}(km/s)$
Mercury	0.39	0.2056	38.7	58.8
Venus	0.72	0.0068	34.9	35.3
Earth	1.00	0.0167	29.3	30.3
Mars	1.52	0.0933	22.0	26.5
Jupiter	5.20	0.0488	12.4	13.7

Most of the NEOs originated in the Main Asteroid Belt, located between Mars and Jupiter, although some of them probably evolved into their current orbits from the reservoir of short-period comets extending beyond Jupiter and into the outer solar system. The range of composition and physical characteristics of asteroid-like NEO spans those found among the Main Belt, though 15% of them probably are derived from cometary reservoirs.

3 Classifications

We have different classifications of the NEOs. Firstly we can classify NEOs by kind and size into three groups:

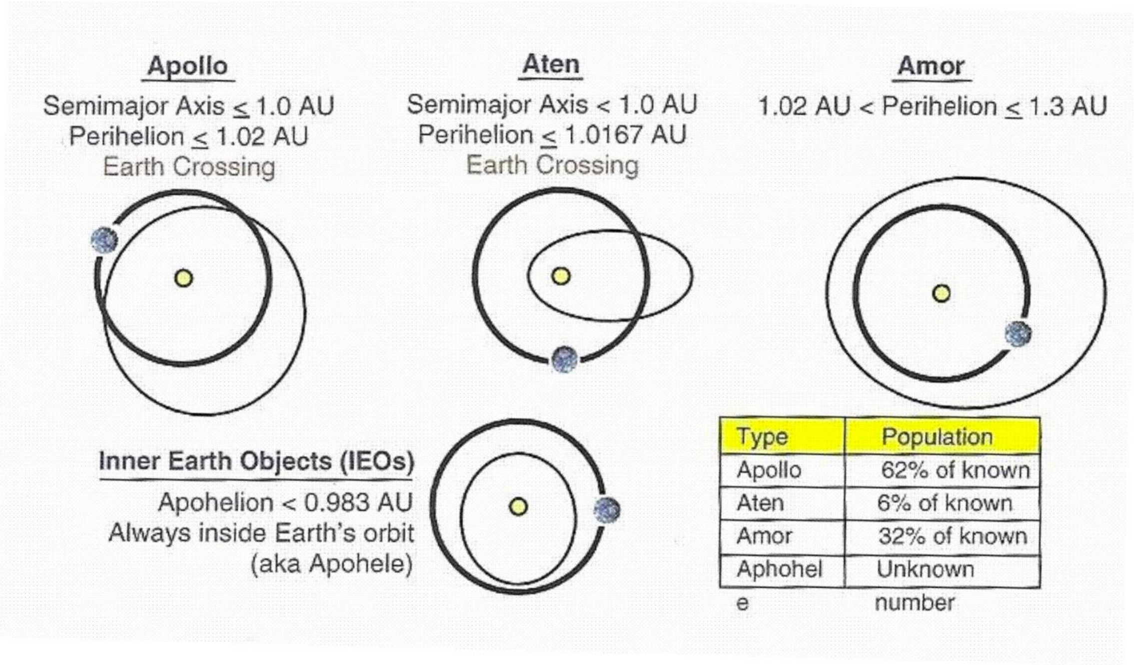
- **Near-Earth meteoroids** are objects in the vicinity of Earth's orbit having a diameter less than 50 meters.
- **Near-Earth asteroids** are objects that have near-Earth orbit, yet far enough from the Sun so that the surface material never evaporates, having a diameter over 50 meters.
- **Near-Earth comets** are objects in the vicinity of Earth's orbit having a nucleus surrounded by tenuous atmosphere (**coma**)

In the group of near-Earth meteoroids there are only meteoroids large enough to be tracked in space. Because the majority of NEO's diameters are estimated using an assumed average albedo for near-Earth asteroids (NEAs), we will not particularly distinguish between near-Earth meteoroids and near-Earth asteroids. Considering this, NEAs are by far the biggest group of NEOs. So far 6610 NEAs are known, ranging in size up to $\sim 32\,km$ (1036 Ganymed), while there are only 84 known near-Earth comets (NECs). NECs are further restricted to include only short-period comets (orbital period P less than 200 years).

NEAs are divided into four subgroups (Apollo, Aten, Amor and Atira) according to their perihelion distance (q), aphelion distance (Q) and their semi-major axes (a) as shown in table 3 and on figure 3. Amors approach but do not cross the orbit of Earth. They have a semimajor axis, $a > 1.0\,AU$, and perihelion $1.017\,AU < q < 1.3\,AU$, between the aphelion of Earth's orbit and inside the perihelion of Mars. Those that actually cross Earth's orbit, Apollos, have $a > 1.0\,AU$ and $q < 1.017$, Earth's aphelion distance. Atens have $a < 1.0$ and $Q > 0.983$, Earth's perihelion distance. Many Atens and all Apollos have orbits that cross orbit of the Earth, so they are a threat to impact the Earth on their current orbits. Amors do not cross the Earth's orbit and are not immediate impact threats. However, their orbits may evolve into Earth-crossing orbits in the future. An object with both a and $q < 0.983AU$, is an Inner Earth Object (IEO) or Apohele asteroid. Apoheles are the subclass of Aten asteroids. However there is currently only 10 known Apoheles, which are according to NASA also categorized as Atira asteroids (named after asteroid 163693 Atira). Atiras are closer to the Sun than Earth and therefore reflect very little light in the direction of the Earth. They are also on the Earth's sky during the daylight and almost impossible to detect. Consequently,

Table 3: Near-Earth Asteroids are divided into four subgroups [5].

Group	**Description**	$q(AU)$	$Q(AU)$	$a(AU)$
Apollo	Earth-crossing NEA with semi-major axis larger than Earth's.	q < 1.017		a > 1.0
Aten	Earth-crossing NEA with semi-major axis smaller than Earth's.		Q > 0.983	a < 1.0
Amor	Earth-approaching NEA with orbits exterior to Earth's but interior to Mars's.	1.017 < q < 1.3		a > 1.0
Atira or IEO	NEA which orbit is contained entirely within the orbit of the Earth.		Q < 0.983	a < 1.0



Type	Population
Apollo	62% of known
Aten	6% of known
Amor	32% of known
Aphohel e	Unknown number

Figure 3: Many Atens and all Apollos have orbits that cross orbit of the Earth while Amors and IEOs only approach Earth and do not cross its orbit [1].

we do not know much about their population. Till the 1st December 2009 there were known 3573 Apollos, 2496 Amors, 531 Atens and only 10 Atiras.

Since the early part of the 20th century, astronomers have recognized that small bodies come in different colors. As observational techniques evolved and the ability to investigate them improved, the number of observable characteristics increased. Sorting objects into meaningful groups is the process of classification or taxonomy. Current asteroid taxonomy[4] is based on the application of statistical clustering techniques to the parameters of color and albedo. The intention of the classification scheme is to reflect the compositional variations and thus their origin and evolution. Today, the alphabet soup of asteroid taxonomy extends to about 12 letters with subtypes numbering up to 26. Of course, the taxonomy too has evolved, and one has to be aware of which system is being used and what the exact definitions are. However, the majority of asteroids fall into the three C, S, and M categories:

- **C-class asteroids**
  C-class asteroids are very dark and non-reflective type of asteroid, gray in color, with a composition believed to be similar to that of carbonaceous chondrites[5] (the "C" stands for carbonaceous). C-class asteroids are the commonest type known (about 75% of all asteroids) and dominate the outer part of the Main Asteroid Belt. They have an

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[4] Asteroid taxonomy has developed on the observational data of the asteroids in the Asteroid Main Belt. Asteroids that have similar color and albedo characterictics are grouped together in a class denoted by a letter or group of letters.

[5] Chondrites are stony meteorites that have not been modified due to melting or differentiation of the parent body. About 80% of all meteorites are chondrites of all types. Carbonaceous chondrites make up less than 5% of the chondrites that fall on Earth.

albedo of 0.03 to 0.09 and a reflectance spectrum that is flat at wavelengths longer than 0.4 micron but shows a feature shorter than 0.4 micron thought to be due to water of crystallization.

- **S-class asteroids**
  A moderately bright, slightly reddish type of asteroid believed to be composed largely of silicate minerals (the "S" stands for silicaceous). This class contains about 17% of asteroids in general. S-class asteroids are quite in the inner part of the Main Asteroid Belt, their proportion decreasing at greater distances from the Sun. They have an albedo of 0.10 to 0.28 and a reflectance spectrum that is flat at wavelengths longer than 0.7 micron. S-class asteroids may be the parent objects of stony-iron meteorites.

- **M-class asteroids**
  A relatively bright and reflective asteroids, made mainly of metallic iron and nickel (the "M" is for metal), typically found in the middle of the Main Asteroid Belt. M-class asteroids are slightly reddish and have featureless reflectance spectra over the range 0.3 to 1.1 microns. They are distinguished from the spectrally similar E-class asteroids and P-class asteroids by their moderate albedo of 0.10 to 0.18. This is the third most populous group.

4 Detecting and Examining NEOs

If we are searching for NEOs it is good to know how do they look like from Earth. The vast majority of NEOs are too small to be seen with naked eye. They are like faint stars or almost unresolvable points of light in a telescope. But how do we distinguish them from stars and other celestial bodies? Since the stars are very far from Earth we hardly detect their proper motions. On the contrary NEOs are very close to Earth and therefore they are moving among fixed stars on the Earth's night sky. Let's take a look at how bright a NEO can be expected from Earth. As we shall see, this will lead to estimate the size of the asteroid and also an estimation of expected impact energy released by a collision with Earth.

4.1 Apparent Magnitude

The brightness of celestial bodies are simply presented in magnitudes. The brighter the object appears, the lower the value of its magnitude:

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{j_1}{j_2} \right) \tag{1}$$

where m_1 and j_1 are magnitude and energy flux of observed body and m_2 and j_2 are magnitude and energy flux of reference body. The relation between a magnitude and an energy flux is logaritmic therefore 5 magnitude difference between two objects means a factor of 100 in the energy flux between the same objects.

The **apparent (or visual) magnitude** (m) of a celestial body is a measure of its brightness as seen by an observer on Earth. The scale upon which magnitude is now measured has its origin in the Hellenistic practice of dividing those stars visible to the naked eye into six magnitudes. The brightest stars were said to be of first magnitude ($m = 1$), while the faintest were of sixth magnitude ($m = 6$), the limit of human visual perception (without the aid of a telescope). Each grade of magnitude was considered to be about 2.5-fold the brightness of the following grade. The modern system is no longer limited to 6 magnitudes or only to visible light. Very bright objects have negative magnitudes. For example, Sirius, the brightest star of the celestial sphere, has an apparent magnitude of -1.4. The modern scale includes the Moon and the Sun; the full Moon has an apparent magnitude of -12.6 and the Sun has an apparent magnitude of -26.73. The Hubble Space Telescope has located stars with magnitudes of 30 at visible wavelengths and the Keck telescopes have located similarly faint stars in the infrared.

4.2 Conversion of Absolute Magnitude to Diameter

The apparent magnitude of an asteroid roughly depends on many physical quantities: size and albedo of the asteroid, distance between the Sun and the asteroid, distance between the Earth and the asteroid and phase angle[6]. Since we are interested in apparent magnitude dependence on the size of the asteroid, we want to fix the other variables at reasonable levels. So we will introduce an asteroid's **absolute magnitude (H)**, which is the apparent magnitude an observer would record if the asteroid were placed 1 AU away from Earth, and 1 AU from the Sun and at a zero phase angle. This is physically impossible, as it requires the observer to be located at the centre of the Sun, but it is convenient for purposes of calculation.

An asteroid is illuminated by the Sun with the incident radiant flux

$$\Phi = \frac{L_\odot}{4\pi r_0^2}\pi R^2 \tag{2}$$

where $L_\odot$ is solar luminosity, r_0 is the distance between Sun and the asteroid ($1\,AU$) and R is radius of the asteroid. Part of the incident flux is absorbed in the asteroid and then emitted over thermal radiation. We will be interested in another part of the incident flux, which is reflected at the asteroid's surface. The question is how this reflection looks like? It is not like reflection on the mirror. We will assume Lambertian reflectance of the surface because of the roughness of the surface. An important consequence of Lambert's law is that when such a surface is viewed from any angle, it has the same apparent radiance (power per unit solid angle per unit projected source area: $L = d^2\Phi/dAd\Omega\cos\theta$). This means, for example, that to the human eye it has the same apparent brightness (or luminance). It has the same radiance because, although the emitted power from a given area element is reduced by the cosine of the emission angle, the size of the observed area is decreased by a corresponding amount. Therefore, its radiance is the same. We can assume that the asteroid is a lambertian scatterer in a shape of disc (perpendicular to the asteroid-observer direction) with a geometric albedo a. Total reflected radiant flux from a illuminated disc with radius R is:

$$\Phi^* = \int I d\Omega$$

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[6] Phase angle is the angle between the light incident onto an observed object and the light reflected from the object. In the context of astronomical observations, this is usually the angle Sun-object-observer.

where I is radiant intensity and $d\Omega$ is infinitesimal solid angle. We also know that $I = I_o \cos\theta$ according to Lambert's law. Due to the symmetry it is good to write infinitesimal solid angle as $d\Omega = dA/r^2 = 2\pi \sin\alpha d\alpha$ (see figure 4). Now we can calculate an integral over α from 0 to $\pi/2$ and get the total reflected radiant flux

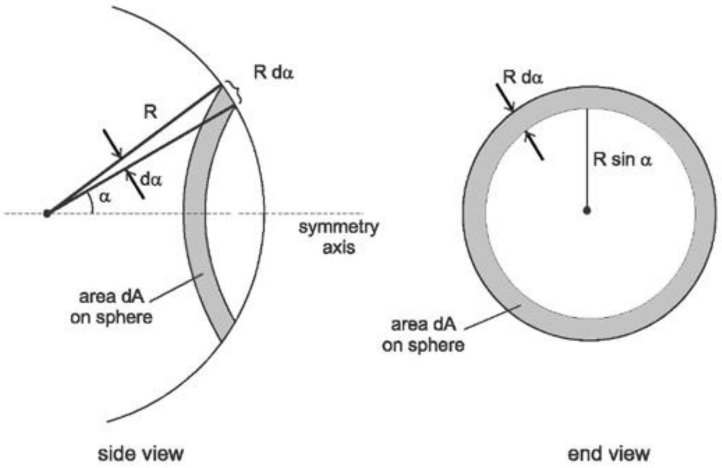

Figure 4: The disc reflects light in a hemisphere. The problem has rotational symmetry (symmetry axis is disc-observer direction). Hence it is convenient to write infinitesimal area as $dA = 2\pi r^2 \sin\alpha d\alpha$ and infinitesimal solid angle as $d\Omega = dA/r^2 = 2\pi \sin\alpha d\alpha$.

$$\Phi^* = \int_0^{\frac{\pi}{2}} (I_0 \cos\alpha) 2\pi \sin\alpha d\alpha = I_0 \pi \int_0^{\frac{\pi}{2}} 2 \sin\alpha d(\sin\alpha) = \pi I_0$$

This is equal to the incident radiant flux (equation 2) multiplied with an albedo a:

$$\frac{aL_\odot}{4\pi r_0^2}\pi R^2 = \pi I_0 \quad \Rightarrow \quad I_0 = \frac{aL_\odot R^2}{4\pi r_0^2} \tag{3}$$

Finally we get disc's radiant intensity in the direction of surface's normal, which we will need later.

Consider that we have a telescope with an aperture area dS. Since almost all objects on the Earth's sky are very far away from us, they appear to be point-like source of light. The telescope collects light of the object through a tiny solid angle $d\Omega = dS/r^2$, where r is the distance between the source of light and the observer. Collected radiant flux is

$$d\Phi = I d\Omega$$

where I is radiant intensity of the object. Considering this and according to the magnitude scale (1), brightness of the sky's objects can be compared to each other as

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{d\Phi_1}{d\Phi_2}\right)$$

where index 1 denotes the first object and index 2 denotes the second object. If we turn the equation, we get

$$\frac{d\Phi_1}{d\Phi_2} = 10^{-\frac{2}{5}(m_1 - m_2)} \tag{4}$$

The ratio of collected radiant flux from the first and second object is

$$\frac{d\Phi_1}{d\Phi_2} = \frac{I_1 d\Omega_1}{I_2 d\Omega_2} = \frac{I_1 dS}{r_1^2}\frac{r_2^2}{I_2 dS} = \frac{I_1}{I_2}\left(\frac{r_2}{r_1}\right)^2 \tag{5}$$

where I_1 and I_2 are radiant intensities and r_1 and r_2 are distances to the first and second objects. Say that the first object is an asteroid reflecting light at the distance 1 AU from the Sun, 1 AU from the observer (Earth) and at the zero phase angle. In this case his apparent magnitude (m_1) is absolute magnitude (H). Observer sees only the light that is reflected exactly in the opposite direction to the incident radiant flux:

$$d\Phi_1 = I_1 d\Omega_1 = I_0 \cos\alpha\, d\Omega_1 \simeq I_0 d\Omega_1$$

where $\alpha \simeq 0$ and I_0 is radiant intensity of the asteroid (3). The second object is a reference star whose magnitude is well known (for example Sun). The detected Sun's radiant flux is

$$d\Phi_2 = I_2 d\Omega_2 = \frac{L_\odot}{4\pi} d\Omega_2$$

Considering this and relation 5, the detected flux ratio is

$$\frac{d\Phi_1}{d\Phi_2} = \frac{a L_\odot R^2}{4\pi r_1^2}\frac{4\pi}{L_\odot}\left(\frac{r_2}{r_1}\right)^2 = \frac{aR^2 r_2^2}{r_1^4} = \frac{aR^2}{r_1^2} \tag{6}$$

where we also take into account that $r_1 = r_2 = 1\,AU$. Now we can combine (4) and (6) and finally get

$$D = 2R = \frac{2r_1}{\sqrt{a}} 10^{\frac{m_2}{5}} 10^{-\frac{H}{5}} = \frac{1340\,km}{\sqrt{a}}\,10^{-\frac{H}{5}} \tag{7}$$

where $m_2 = -26.74$ apparent magnitude of the Sun, H is asteroid's absolute magnitude and a is asteroid's albedo. We have showed that the diameter of an asteroid can be estimated from its absolute magnitude (H). The lower the H value, the larger the size of the object. However, this also requires that the asteroid's albedo be known as well. Since the albedo for most asteroids is not known, an albedo range between 0.05 to 0.5 is usually assumed (figure 5) and these results in a range for the diameter of the asteroid.

4.3 Expected Impact Energy

The energy released by a collision is, for all practical purposes, entirely kinetic, expressed in the usual form

$$E = \frac{1}{2} m v_i^2$$

The impact energy is poorly defined in most cases due to substantial uncertainty in the object's mass m. In the absence of an accurate mass determination, the mass must be inferred from the size, shape and density of an object, but for all potential impactors discovered to date, none of these information has been directly available, and this trend is likely to continue because the vast majority of potential impacts are associated with poorely observed objects.

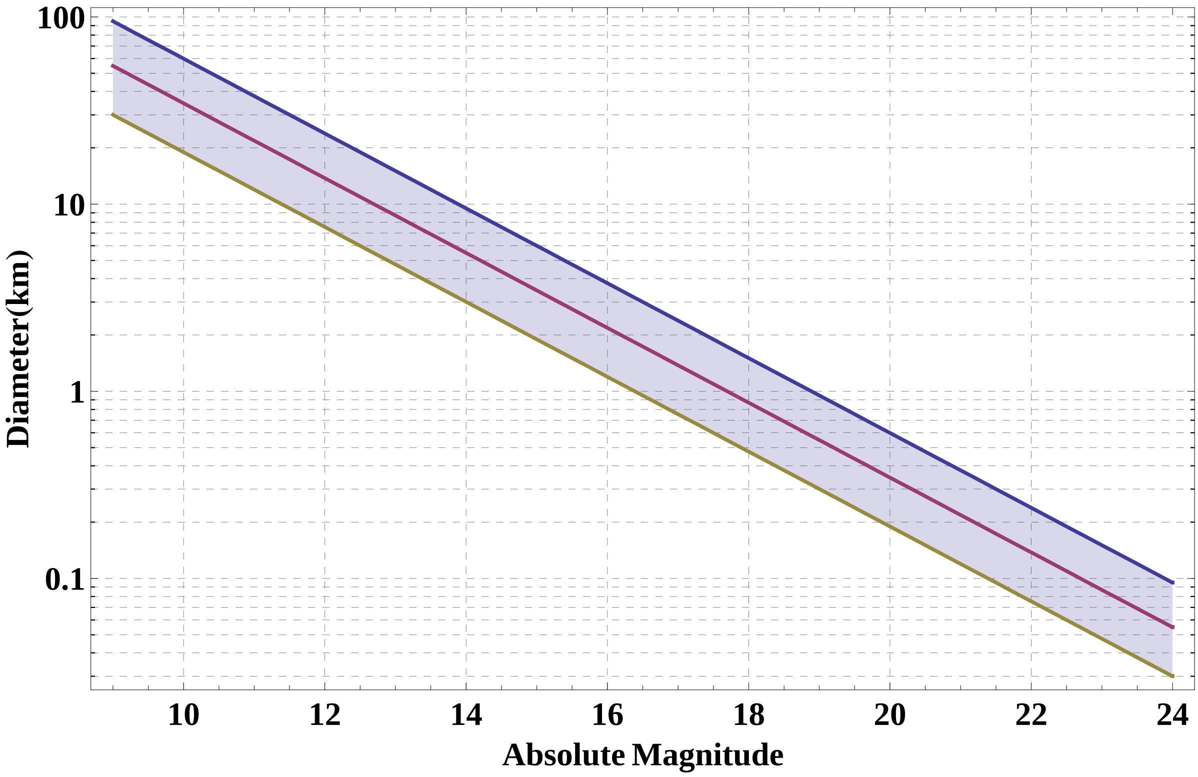

Figure 5: Conversion of Absolute Magnitude to Diameter. We assumed $a = 0.05$ at the blue line, $a = 0.15$ at the red line and $a = 0.5$ at the yellow line. Colored part of the graph covers asteroid's reflectances from 0.05 to 0.5.

Typically, the only information available is the absolute magnitue H of the object, which relates its intrinsic brightness. In such circumstances, we are forced to assume a homogeneus spherical object with density ρ, diameter D and mass given by

$$m = \frac{\pi}{6}\rho D^3$$

Given H and the albedo a, the diameter can be computed from 7. The principal source of error in the size of the object generally arises from the albedo uncertainty. However, the computed value of H can easily be wrong by a half magnitude or more since several simplifying assumptions were made about the object's phase relation, shape of the body, their surface reflection,... For asteroid's density ρ is assumed the value of $2.6\,g/cm^3$. The mass estimate (and similar expected impact energy) is somewhat more rough than the diameter estimate, but generally will be accurate to within a factor of three.

On the other hand, the impact velocity v_i is available to high precision since the impact trajectory is known for any given impact. From the total energy (kinetic and potential) conservation law follows

$$v_i^2 = v_\infty^2 + v_{escape}^2$$

where v_∞ is relative velocity at atmospheric entry neglecting the acceleration caused by the Earth's gravity field (often called the hyperbolic excess velocity) and $v_{escape} \sim 11.2\,km/s$ is the Earth escape velocity. As we see, the minimum velocity of any object entering the Earth's atmosphere is equal to the Earth's escape velocity. So even relatively slow-moving NEOs can have quite significant energy when they hit (blue line on graph 6)[7]. To estimate the maximum impact velocity we will imagine an asteroid with extremely high relative velocity to Earth, for example Sun's escape velocity and will collide it with Earth in the front direction. Earth's orbital velocity is $\sqrt{GM_{\odot}/r} \approx 30\, km/s$ and Sun's escape velocity is $\sqrt{2GM_{\odot}/r} \approx 42\, km/s$, so the maximum velocity impact would be around 72 km/s. Of course, such objects would have to orbit around the Sun in opposite direction as the Earth (and the vast majority of other objects) and are extremely rare among asteroids but quite frequent among meteoroids. Those impacts would unleash very high energies represented on graph 6 with red line. Most scientists consider 1 km as the size large enough for an impact to present a global threat to human survival.

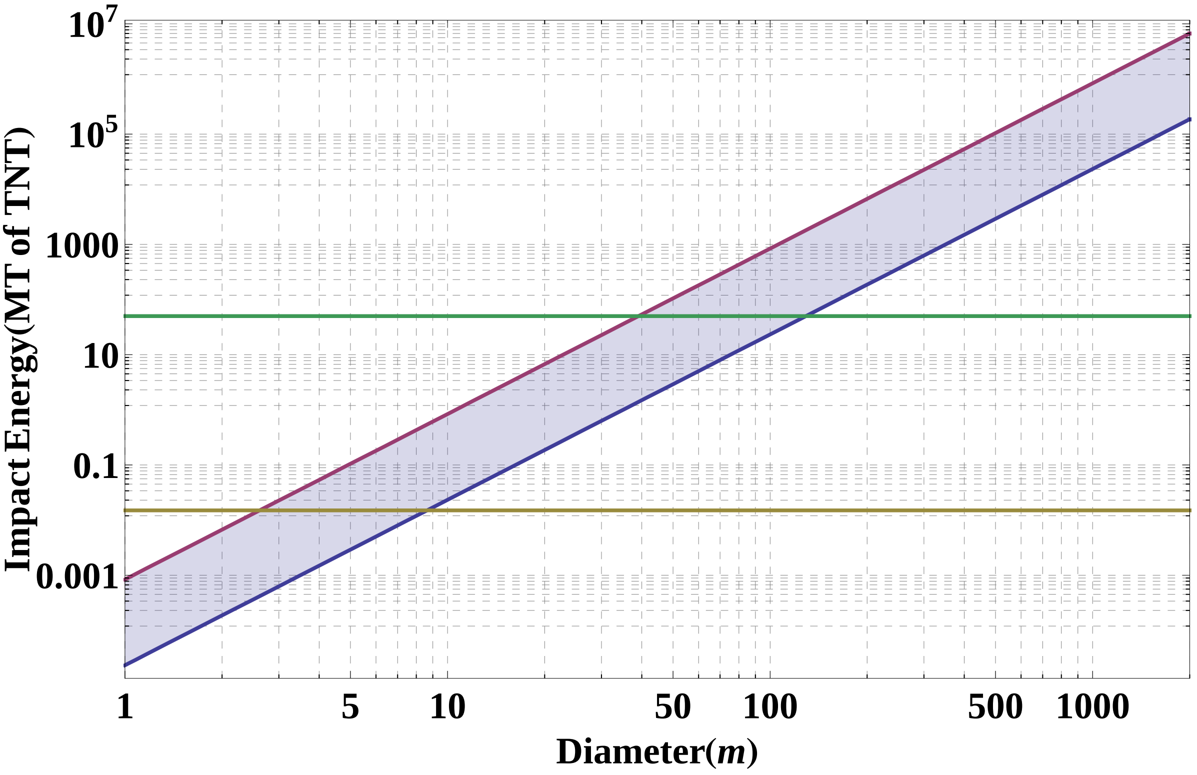

Figure 6: Blue line is representing minimum impact energy at a given diameter with assumed impact velocity 12 km/s. Red line is presenting maximum impact energy at a given diameter with assumed impact velocity 72 km/s. We have also assumed that the object is always spherical with density $\rho = 2.6\, g/cm^3$. For comparison, there is also the yellow line showing the energy released by the atomic bomb dropped on Hiroshima and the green line representing the most powerful nuclear weapon ever detonated – Tsar bomb.

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[7] One ton (T) of TNT is a unit of energy equal to 4.184 GJ, which is approximately the amount of energy released in the detonation of one ton of TNT. For comparison, the atomic bomb dropped on Hiroshima released approximately 15 kT of TNT. The most powerful nuclear weapon ever detonated released 50 MT of energy (Tsar Bomb, detonated in 1961 by Soviet Union).

4.4 Population

It is difficult to quote the definitive size of the NEO population. Search programs are constantly adding to the inventory, but there are inherent limitations in search techniques. Consider setting out to count the number of NEOs. First one can only look for them at night. At any time, one can only search half the sky. Then there are limitations in how much sky one can cover in one night, controlled by the telescope field of view and the recording instrumentation. The realities of weather and equipment performance further hinder the search...

To date, search programs have found 6694 NEOs of all sizes 7. 84 of them are near-Earth comets (NECs), the rest are NEAs. 800 of the discovered NEAs are one kilometer or more in diameter. The biggest objects appear brightest and are most easily found. There are 30 NEOs as large as 5 km across, arround 300 objects have been cataloged that are larger than 2 km, and NEO catalogs are nearly complete at this size. Catalogs are known to be incomplete for objects smaller than 2 km, but by knowing how much area of the sky has been searched and how sensitive these searches have been, it is possible to estimate how many objects are left to find. An article [9] estimate that there are 855 ± 110 of NEOs larger than a kilometer. A more recent Ph.D. thesis by J. Scott Stuart estimates that there are about 1100 total NEOs larger than 1 km in diameter and up to 85,000 NEOs larger than 100 m.

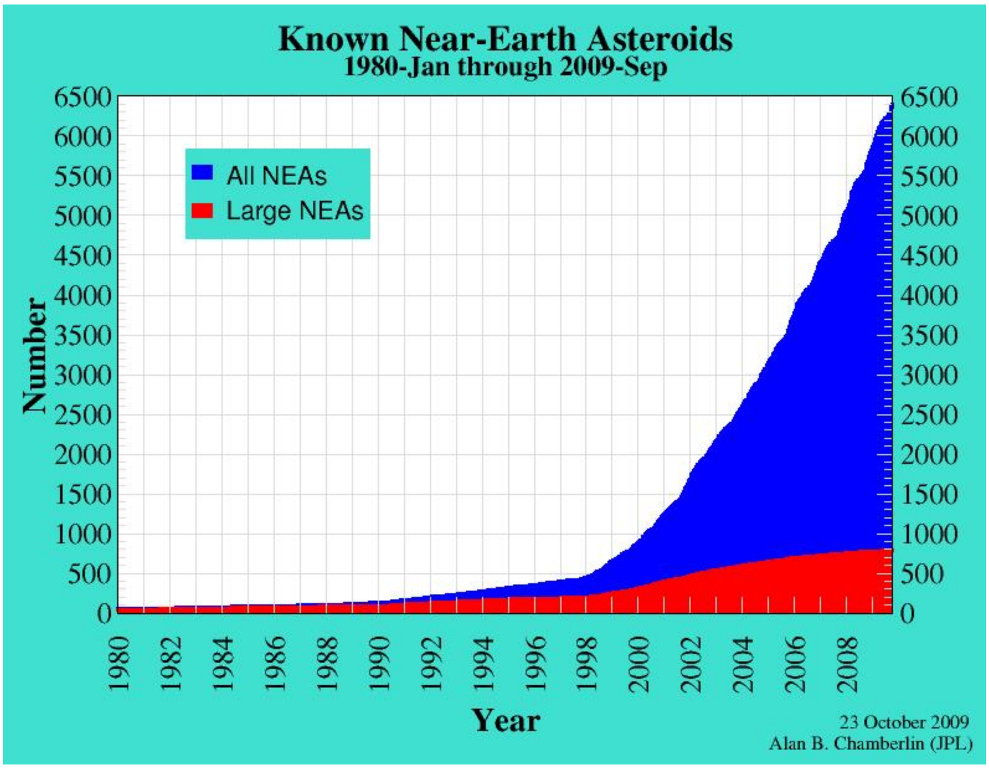

Figure 7: The chart above shows the cumulative total known near-Earth asteroids versus time. The blue area shows all near-Earth asteroids while the red area shows only large near-Earth asteroids (those with diameters roughly one kilometer and larger)[5].

5 Earth Collision Hazards

Most objects colliding today with Earth are small and harmless. Fragments that are a meter to a centimeter in size appear as bright bolides in the sky and can deliver meteorites to the ground, though these are essentially harmless. But how safe we really are?

5.1 Potentially Hazardous Asteroid

Potentially Hazardous Asteroids (PHAs) are currently defined based on parameters that measure the asteroid's potential to make threatening close approaches to the Earth. Specifically, all asteroids with a minimum orbit intersection distance (MOID) of 0.05 AU or less and an absolute magnitude (H) of 22.0 or less are considered PHAs. In other words, asteroids that can not get any closer to the Earth (i.e. MOID) than 0.05 AU (roughly 7,480,000 km or almost 20 lunar distances) or are smaller than about 150 m in diameter (i.e. H = 22.0 with assumed albedo of 13%) are not considered PHAs.

This "potential" to make close Earth approaches does not mean a PHA will impact the Earth. It only means that there is a possibility for such a threat. By monitoring these PHAs and updating their orbits as new observations become available, we can better predict the close-approach statistics and thus their Earth-impact threat. So far they have discovered 1078 PHAs, 146 of them are one kilometer or more in diameter.

5.2 Collision Probability

When a NEO is first discovered, astronomers initially trace only a short piece of its orbit as measured over a few hours or even over a few weeks. With each new NEO discovery, astronomers wish to assess whether the object poses any immediate or future impact threat. Orbit calculations for most objects can be made reliably for many decades into the future, but of course if only a tiny part of the orbit has been observed, the extrapolation into the future becomes increasingly uncertain. Sometimes that extrapolation shows that the Earth itself resides within the overall uncertainty region for an NEO's future position. If the cross section of the Earth occupies 1/10,000th of this space, then there is a 1 in 10,000 chance of an impact with the Earth. Even though headlines may proclaim the end of the world, statistically speaking, the odds are actually 10,000 to 1 in our favor that continued observations refining the orbit will show a collision probability is decreased and ultimately ruled out.

Imagine that there is a PHA, which trajectory is perfectly defined and at some point intersects Earth's path around the Sun. What is the probability for collision in one Earth's tour arround the Sun? If the PHA has a semimajor axis a, eccentricity ϵ and diameter D, its trajectory around the Sun describe a ring-shape volume

$$\begin{aligned} V &= \pi \left(\frac{D}{2}\right)^2 C(a,\epsilon) \\ V &= \pi \left(\frac{D}{2}\right)^2 a\pi \left(3(1+\sqrt{1-\epsilon^2}) - \sqrt{(3+\sqrt{1-\epsilon^2})(1+3\sqrt{1-\epsilon^2})}\right) \end{aligned}$$

where $C(a,\epsilon)$ is ellipse's circumference (considering Ramanujan's approximation). If the PHA's trajectory is perpendicular to the Earth's orbit then the cross section of the Earth occupies volume nearly $2R_{\oplus}\pi(D/2)^2$, where $R_{\oplus}$ is Earth's radius. The collision probability is

$$\mathcal{P} \sim \frac{2R_{\oplus}}{C(a,\epsilon)}$$

Thus for a typical PHA with $a \sim 1\,AU$ and $\epsilon \sim 0.2$ the collision probability yields $\mathcal{P} \sim 10^{-5}$. This result tells us that although we have assumed that in our case the PHA's trajectory intersects the Earth's orbit[8], the probability of collision is very low. One could also say the Earth is very small target in the universe. Usually it is not difficult to determine where the PHA's orbit intersects the Earth's orbit. Since the Earth's orbit is very well known, the main error of the intersection "point" is due to uncertainty of the PHA's orbit. This could be refined with careful observations over a longer period.

One of the main problem in predicting the likelihood of collision is hidden in the timing. Suppose that we know the intersection point of the Earth's and PHA's orbits. The Earth travels around the Sun with a speed $\sim 30\,km/s$ and the PHA travels with an even greater speed because of higher eccentricity of its orbit. If the PHA is supposed to collide with Earth, it should not be delayed or too fast. Earth actually crosses the point of intersection within $2R_{\oplus}/v_{\oplus} \sim 7.1$ minutes. Therefore the PHA's velocity prediction must be as precise as possible. Strictly speaking, accuracy of speed must be better than $0.1\,m/s$. Possible delays or overtaking from our projection are added cumulatively to error and this is a great problem in predicting of collision.

PHAs are relatively small bodies and consequently their trajectories are also effected by other dissipative nongravitational forces. In this context I would like to mention the Yarkovski force, which is important for bodies ranging in size from meters to several kilometers. In particular case it is not easy to estimate how much is the orbit affected by the Yarkovsky effect. Probably it would be necessarily to place a transmitter on the PHA for sufficiently accurate tracking.

5.2.1 Yarkovski Effect

Consider a rotating body heated by the Sun[9]. Because of thermal inertia, the afternoon hemisphere is typically warmer than the morning hemisphere, by an amount $\Delta T << T$. Let us assume that the temperature of the morning hemisphere is $T - \Delta T/2$, and that of the evening hemisphere $T + \Delta T/2$. The radiation reaction upon a surface element dA, normal to its surface, is $dF = 2\sigma T^4 dA/3c$. For a spherical partical of radius R, the Yarkovski force in the orbit plane due to the excess emission on the evening side is

$$F_Y = \frac{8}{3}\pi R^2 \frac{\sigma T^4}{c} \frac{\Delta T}{T} \cos\phi$$

where σ is the Stefan-Boltzmann constant and ϕ is the particle's obliquity, that is the angle between its rotation axis and orbit pole. The reaction force is positive for an object that rotates in the prograde direction, $0° < \phi < 90°$, and negative for an object with retrograde rotation, $90° < \phi < 180°$.

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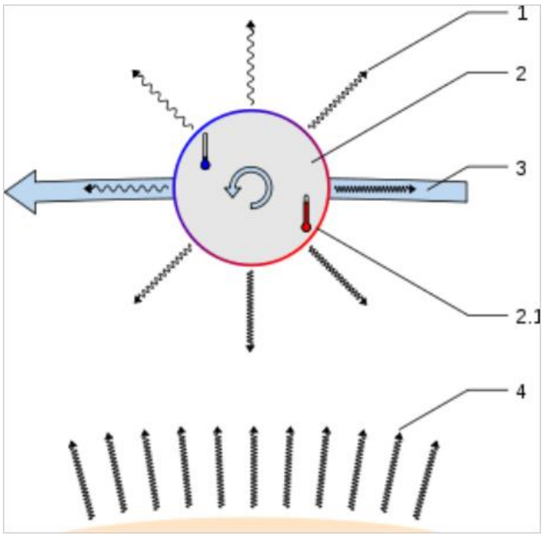
[8] The vast majority of NEOs come in the vicinity of Earth's orbit but do not intersect it.

[9] On a distance 1 AU from the Sun the force of the solar wind on a body could be neglected since it is much smaller than radiation force on the same body.

Sketch showing Yarkovski effect:
1 Radiation from asteroid's surface
2 Prograde rotating asteroid
2.1 Location with "Afternoon"
3 Asteroid's orbit
4 Radiation from Sun



5.3 Quantifying the Impact Risk

There have been numerous detections of asteroid orbital solutions that lead to an Earth impact, usually several decades in the future and with very remote impact probabilities. Indeed, detections are occurring at such a frequent rate that they are now becoming routine. This situation has created the need among impact specialists for a means of systematically measuring the importance of so many cases in ordor to put them in their proper context. Any scheme designed to measure the importance of a potential impact should consider at least three general factors, each of which have roughly equal importance.

Impact Date. The time until the event determines the time available to react to the threat, which strongly influences the appropriate response. Clearly, high probability impacts taking place at some months, years, decades, centuries, or millennia in the future would each lead to very different threat mitigation strategies and levels of public interest.

Impact Energy. The consequences of the impact, should it occur, are clearly a key consideration in determining the proper pofessional and public response. Impacts have the potential for local, regional, or global destruction. Other impacts, where the object is unlikely to cause any surface damage at all, may yet be of considerable scientific importance.

Impact Probability. The likelihood that the impact will actually occur is also of obvious importance. In particular, the significance of a predicted impact event relative to the mean impact frequency can indicate the level of interest appropriate to the situation.

There has been established the **Torino Scale** as a "*tool for public communication and assessment for asteroid and comet impact hazard prediction in the next century*". The Torino Scale present a clear and very simple measure of a hazard posed by a potential collision using a 10-point integer scale. Currently, there is only one object labeled with number 1, which means that a pass near the Earth is predicted but new telescopic observations very likely will lead to re-assignment to Level 0. However, the simplicity of the Torino Scale makes it relatively unsuited for use by specialists in categorizing large numbers of events and in prioritizing objects for observation analysis. So there has been introduced also another system, the **Palermo Scale**, that smoothly characterizes impact hazards across the entire range of impact dates, energies and probabilities. The Palermo Scale is more complicated and intended for astronomers [6].

6 Conclusion

NEOs come closer to Earth than any other planetary bodies. Compared to other objects in space these are relatively small bodies but in case of collision with Earth it could be as well the end of our civilization. Most scientist consider 1 km as the size large enough for an impact to present a global threat to human survival. Current search efforts have as their most immediate goal to find all objects larger than 1 km. Organized, telescopic search programs for NEOs operate worldwide. The most successful NEO search programs are Catalina Sky Survey, LINEAR and Spacewatch. There is also a contribution of 14 NEOs discovered by Slovenian search program PIKA at Črni Vrh Observatory [7]. The value of the searches is to change our knowledge from probably being safe to being highly certain about any threat from impacts for many generations. To estimate the potential for any NEO to collide with Earth, it is imperative to have an accurate assessment of the number and location of this population. Also it is important to know the nature of orbit for each object because it is directly related to the object's velocity relative to Earth and its motion around the Sun. The active asteroid search programs are designed to inventory the population of objects that may impact Earth. Upon discovery of new NEO, its orbit is determined and its future orbital evolution is projected by computer simulations. If there is a potential threat of its impacting Earth, a call for follow-up observations is made, and the threat is evaluated carefully. The existence of PHAs is monitored closely, worldwide. Although the probability of Earth impact by NEOs is rather low, it should be considered seriously on large time-scales.

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