Donald R. Satterlund[1]

Roger C. Chapman

Department of Forestry and Range Management

and

Richard D. Beach[2]

Program in Environmental Science
Washington State University
Pullman, Washington 99164

Modeling the Daily Temperature Cycle

Abstract

A simple model has been developed that can be used to predict the mean daily temperature cycle, by month, provided the mean daily minimum and maximum temperatures are known. Only one set of monthly coefficients is needed for all inland stations in the Pacific Northwest, from Butte, Montana, to Olympia, Washington; but a separate set of coefficients is needed for coastal stations influenced by summer fogs and diurnal land-sea breezes. Tests of the model in predicting mean hourly temperatures at three-hour intervals for all months at seven inland stations yielded a RMSE of 0.96°F (0.6°C) and a maximum absolute error of 3.6°F (2.0°C). Similar tests at two coastal stations yielded a RMSE of 0.35°F (0.2°C), with a maximum absolute error of 1.0°F (0.6°C). The model is based on the equation describing a Pearson type III distribution.

Introduction

Temperature is a primary driving variable in nearly all physical and biological processes —the melting of snow or freezing of water, partitioning of net radiation, plant phenology and growth, and many others. Changing seasonal phenomena may seem self-evident responses to changing temperatures, but the shorter diurnal cycles are not so obvious. Nevertheless, the diurnal cycle may be considered a small scale representation of the annual cycle, and the diurnal range in mean hourly temperatures in a given month may approach the yearly range in mean monthly temperatures.

Physical and biological processes are similarly responsive to these diurnal variations. Snow that is frozen in the morning may be melting in the afternoon. Evapotranspiration may increase by 15-25 percent from morning to afternoon (Gay and Fritschen 1979). McCutchan (1976) cited the need for hourly temperature data for use in a simulation model in forest fire management. These observations indicate the importance of the diurnal temperature cycle in energy exchange and ecosystem processes, particularly in mountainous areas where there are pronounced differences by slope direction and inclination. If we are to be able to model these processes and relate land management responses to specific environments, we must know the daily temperature cycle.

Air temperature is one of the most easily measured of all environmental parameters, yet hourly or continuous temperature data are not readily available in most places.

[1]The senior author is also affiliated with the Program in Environmental Science at Washington State University.

[2]Present address: 520 NW 4th Avenue, Corvallis, Oregon 97330.

Maximum and minimum thermometers are standard instruments in any climatic station, but thermographs that yield hourly or continuous temperature data are usually found only in well equipped first order stations and research facilities.

The lack of data describing the daily temperature cycle has impeded efforts by the senior author to develop estimates of net radiation and its partitioning on mountain slopes. Therefore, we set out in an attempt to develop a simple, general model of the daily temperature cycle that would require only mean minimum and maximum temperature data. This paper presents the results of that effort.

An examination of the literature revealed three basic approaches to the problem of estimating the diurnal temperature cycle: (1) correlation-regression models (Madden and Shea 1978); (2) simple sine curves (Johnson and Fitzpatrick 1977a); and most commonly, (3) reproduction of measured cycles using Fourier series (McCutchan 1976), or a combination of Fourier analysis and sine curves (Johnson and Fitzpatrick 1977b). None seemed fully satisfactory for lack of accuracy, lack of generality, or lack of available data.

Methods

Mean dry bulb air temperature at three hour intervals, by month, were available from 48 climatic stations scattered across two time zones of the Pacific Northwest (Pacific Northwest River Basin Commission 1968). They were gathered over the 10 year period 1949-1958 mostly from airports, but include four mountain passes and two offshore islands in Washington, Oregon, Idaho, and western Montana (Fig. 1).

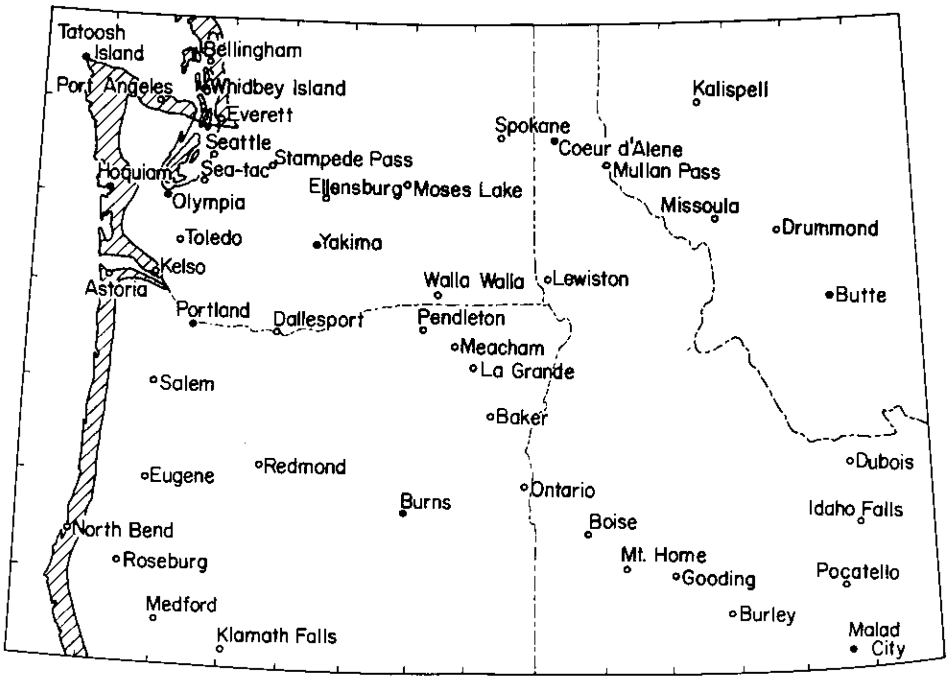

Figure 1. Interior and coastal (hatched) regions of daily temperature cycles. Solid circles indicate stations used for testing model, open circles are stations used as data sources to derive the model.

Normalizing the Data

To reveal whether all stations exhibited a similar temperature pattern, the data from all stations were normalized to the same amplitude and time base to reveal the pure form of the cycle.

Normalization of amplitude was achieved by expressing the temperature at any given time as a decimal fraction of the range. The mean daily instantaneous minimum temperature, by month, was assigned a value of 0, and the corresponding maximum, 1.0. This procedure removed variation in apparent pattern due to widely different diurnal temperature ranges among different months and locations. As the mean instantaneous minimum temperature is always less than the mean three hour minimum (and the maximum greater), the range of the data was increased by 10 percent to correspond with measured instantaneous mean minimums and maximums (Pacific Northwest River Basins Commission 1969). Thus, if the mean three hour minimum temperature was 40°F, and the corresponding maximum 70°F, the range is 30°F and a 10 percent increase of 3°F results in a mean instantaneous minimum of 38.5°F and a maximum of 71.5°F.

In addition to normalizing the temperature amplitude, it was necessary to express all time data in terms of a common time, rather than local time. All clock times were therefore converted to solar time for the appropriate station longitude, using the equation of time (Frank and Lee 1966).

The normalization procedure removed all variability in amplitude and timing in the temperature data except that due to differences in form of the diurnal cycle.

Testing the Data for Uniformity of Form

All data, reduced to a common amplitude and time base, were plotted on graph paper (12 months x 48 stations = 576 curves) to see if they revealed a common form. Visual inspection seemed to confirm a single characteristic shape, resembling a somewhat distorted sine curve for all stations. However, the period of high temperature was broader, and the maximum occurred later during the summer than winter months, as would be expected from the longer period of daylight in summer. Therefore, the curves were segregated by month.

When segregated by month, it became evident that the curves from coastal areas seemed to differ from stations in the interior, exhibiting summer maximums earlier in the day with shorter periods of high temperature. Nine coastal stations exhibiting this pattern were segregated from the rest for separate analysis.

At this stage in the screening process, the data from nine stations were reserved for later use in testing the fit of any model to be developed. The stations, seven from the interior and two from the coast, were selected in a stratified random manner by putting slips with the names of all stations from each group in a single state into a hat and drawing them out. Interior stations reserved were two of eleven from Idaho, one of four from Montana, two of thirteen from Oregon, and two of eleven from Washington. Two of nine coastal stations from Oregon and Washington combined were also reserved. The data from these stations were excluded from those used to develop a model, being reserved to provide independent tests to fit to specific stations.

The remaining data, segregated into interior (32 stations) and coastal (7 stations) sets, were then tested to establish the degree of uniformity of the daily temperature cycle during each month. The population mean and standard deviation of the normalized temperature at each of eight 3-hour intervals throughout the day, for each month, was determined. Only two (of 96 populations) exhibited standard deviations exceeding 0.1 (10 percent) of the mean daily temperature range in each set. From this analysis, we were satisfied that a single model, using the same coefficients, should adequately represent the mean daily temperature cycle for all stations within a given region in any month.

The model remained to be derived, but the goal of a single general model that would be sufficient to describe the mean daily temperature cycle for any station appeared feasible of being met.

Choosing a Model

Inspection revealed that a simple sine curve obviously did not describe the normalized temperature cycle. Trials by Beach (1979) suggested that attempts to develop a model based on Fourier series using only mean monthly minimum and maximum temperature observations were not likely to be fruitful without a heroic number of calculations. For example, McCutchan (1976) required the solution of 30 regression equations to obtain the coefficients for his model from a much stronger data base.

Chapman, observing the fixed upper and lower limits of the normalized curves and the regular seasonal variation in their form, established that a Pearson type III distribution would encompass the range in variation expressed by the data. The Pearson type III model

$$Y = e^{-x\gamma}\left(1 + \tfrac{x}{a}\right)^{\gamma a} \tag{1}$$

could be modified to yield the temperature at any time, t, provided the coefficients γ, x, and a were known. In modified form to yield temperature:

$$T_t = T_m + T_r \left[e^{-\gamma t}\left(1 + \tfrac{t}{a}\right)^{\gamma a}\right] \tag{2}$$

where T = temperature

t = time, hours before or after time of temperature maximum; subject to $t < o, |t| \leq a$ and $t > o, |t| \leq 24 - a$.

m = minimum

r = range

a = length of period of rising temperature, hours

γ = an empirically derived coefficient, and

e = base of the natural logarithm (2.718 . . .).

The mean monthly minimum and maximum temperature, and hence the temperature range, are obtained from climatic records of the station of interest. The times of maximum and minimum temperatures were determined by inspection of the mean normalized temperature graph, and the value of the coefficient, "a", determined by subtracting the time of the minimum temperature from the time of the maximum temperature. The value of the coefficient, t, is assigned as the time of interest before (−t) or after (t), the time of maximum temperature. Units involving time were expressed to the nearest 0.1 hours.

The value of the γ coefficients were determined by fitting equation (1) to the mean normalized temperature curve for interior or coastal stations for each month by the method of successive approximation using intervals of 0.01. As the analysis progressed, it soon became apparent that separate values of γ were necessary when $t < 0$ and when $t \geq 0$. The equation is discontinuous at time $t = 24 - a$. A best fit was accepted when the root mean square deviations of calculated normalized temperature from the observed mean normalized temperature curve for the hours 1, 4, . . . 19, and 22 hours were minimized.

Results

The model of equation (1) fits the mean diurnal normalized temperature curve for all months and both data sets very well. The correlation coefficient between estimated and normalized mean temperature exceeded 0.90 and was significant at or better than the 0.01 probability level in all cases. We had demonstrated that a single, simple model could describe the mean daily temperature cycle for any month for either interior or coastal station data sets. Slightly different coefficients were required to fit interior and coastal patterns. The values of each coefficient for each month and station set are shown in Table 1.

TABLE 1. Coefficients for the equation $T_t = T_m + T_r\,[e^{\gamma t}(1+\beta^{\gamma a})]$ to estimate temperature in the Pacific Northwest.

Month	Hr. of maximum T		a (hrs.)		γ, t < 0		γ, t > 0	
	Inland	Coast	Inland	Coast	Inland	Coast	Inland	Coast
Jan	13.5	13.2	7.2	7.2	0.50	0.62	0.55	0.65
Feb	14.3	14.0	8.0	8.0	0.37	0.40	0.62	0.68
Mar	14.5	14.2	8.6	8.3	0.28	0.28	0.60	0.65
Apr	14.7	14.2	9.9	9.4	0.20	0.20	0.62	0.72
May	14.7	14.2	10.4	10.2	0.20	0.20	0.62	0.72
Jun	14.8	14.2	11.0	10.7	0.20	0.22	0.62	0.72
Jul	15.0	14.2	11.3	10.6	0.22	0.24	0.68	0.70
Aug	14.7	14.2	10.4	9.8	0.24	0.24	0.60	0.75
Sep	14.5	14.2	9.4	9.0	0.23	0.20	0.64	0.78
Oct	14.1	13.8	8.5	8.2	0.24	0.24	0.74	0.82
Nov	13.6	13.2	7.8	7.6	0.36	0.42	0.66	0.78
Dec	13.5	13.0	7.2	7.2	0.48	0.60	0.66	0.66

It is one thing to force fit a model to the mean curves from which the coefficients of the model were derived, but it remained to be seen whether the model and the same coefficients could be used to reproduce the mean daily temperature curve, by month, for individual stations independent of those used to derive them. Accordingly, the model of equation (2) using the coefficients of Table 1 was used to predict mean dry bulb temperatures at three hour intervals (1, 4, . . . 22 hours), by month, at each of the nine stations reserved for independent tests of the model.

The independent tests demonstrate an accurate fit of estimated to actual temperature for all months and all stations. The maximum deviation found for all 864 station-hour-months was 3.6°F (2°C) at Portland, Oregon, at 10 a.m. in July, and the maximum root mean square deviation of the eight hourly estimates of the mean daily cycle was 1.83°F (1°C) for Burns, Oregon, in July, when the observed mean daily temperature amplitude was 35.9° (20°C). Table 2 shows the statistical data of degree of fit for each station month used in this independent test. The data for five selected months for the station with the poorest hourly fit (Portland, Oregon) and the best overall fit (Coeur d'Alene, Idaho) are presented in Figures 2 and 3. When all test data for the inland stations were combined (8 hours x 12 months x 7 stations = 672 station-hour-months), the root mean squared error (RMSE) in estimated temperature was 0.96°F

TABLE 2. Statistics of fit of estimated to measured mean temperatures from nine stations, for 8 hrs. each day.

Station	Month	RMSE(°F)	Max. absolute error(°F)	r^2
Interior				
Coeur d'Alene, Idaho	1	0.27	0.5	.9929
	2	0.24	0.4	.9967
	3	0.43	0.9	.9946
	4	0.45	0.8	.9744
	5	0.67	1.2	.9911
	6	0.67	1.0	.9910
	7	1.04	1.6	.9880
	8	1.00	2.0	.9873
	9	0.90	1.7	.9893
	10	0.59	1.3	.9931
	11	0.53	1.1	.9811
	12	0.30	0.5	.9807
	yr.	0.65	2.0	.9984
Malad City, Idaho	1	0.79	1.7	.9783
	2	0.77	1.4	.9857
	3	0.97	1.6	.9768
	4	1.38	2.4	.9775
	5	1.69	3.3	.9665
	6	1.45	2.9	.9832
	7	1.41	2.5	.9860
	8	1.74	2.5	.9800
	9	1.77	2.7	.9804
	10	1.02	1.6	.9938
	11	1.13	1.8	.9813
	12	0.94	2.0	.9615
	yr.	1.30	3.3	.9954
Butte, Montana	1	0.83	1.1	.9803
	2	0.56	1.0	.9963
	3	0.74	1.4	.9879
	4	0.51	1.0	.9976
	5	0.88	1.8	.9885
	6	0.66	1.4	.9946
	7	1.37	2.1	.9923
	8	1.17	1.8	.9897
	9	0.51	0.8	.9978
	10	0.77	1.6	.9957
	11	0.86	1.4	.9933
	12	1.06	1.7	.9852
	yr.	0.87	2.1	.9981
Burns, Oregon	1	1.19	2.0	.9613
	2	1.19	2.0	.9819
	3	0.44	0.7	.9954
	4	0.71	1.2	.9936
	5	0.73	1.0	.9926
	6	1.02	1.8	.9900
	7	1.83	2.5	.9752
	8	1.37	2.9	.9822
	9	1.73	3.4	.9737
	10	1.69	3.2	.9742
	11	1.04	2.1	.9856
	12	1.08	1.8	.9742
	yr.	1.24	3.4	.9951
Portland, Oregon	1	0.36	0.6	.9776
	2	0.36	0.7	.9921
	3	0.50	0.8	.9956
	4	0.78	1.8	.9883
	5	0.82	1.9	.9894
	6	0.91	2.1	.9873
	7	1.73	3.6	.9640

TABLE 2. (Continued)

	8	1.39	3.4	.9747
	9	0.95	2.2	.9854
	10	0.63	1.3	.9929
	11	0.46	0.7	.9901
	12	0.34	0.7	.9854
	yr.	0.88	3.6	.9959
Olympia, Washington	1	0.34	0.5	.9861
	2	0.26	0.5	.9956
	3	0.30	0.6	.9980
	4	0.27	0.4	.9995
	5	0.62	1.0	.9941
	6	0.73	1.4	.9918
	7	1.12	2.2	.9894
	8	0.83	1.5	.9941
	9	0.47	0.8	.9975
	10	0.32	0.5	.9971
	11	0.39	0.6	.9908
	12	0.30	0.5	.9899
	yr.	0.56	2.2	.9982
Yakima, Washington	1	0.51	0.8	.9904
	2	0.62	1.0	.9873
	3	0.86	1.6	.9872
	4	0.86	1.9	.9941
	5	1.14	2.9	.9861
	6	0.75	2.0	.9936
	7	1.27	2.6	.9937
	8	1.30	2.5	.9884
	9	1.42	2.4	.9829
	10	1.09	1.9	.9865
	11	0.52	0.9	.9927
	12	0.52	0.8	.9911
	yr.	0.96	2.9	.9971
Coastal				
Hoquiam, Washington	1	0.26	0.4	.9912
	2	0.32	0.5	.9896
	3	0.38	0.8	.9976
	4	0.50	0.9	.9930
	5	0.50	1.0	.9914
	6	0.34	0.6	.9947
	7	0.33	0.5	.9973
	8	0.50	0.8	.9702
	9	0.49	1.0	.9946
	10	0.45	0.9	.9965
	11	0.55	0.9	.9841
	12	0.17	0.3	.9960
	yr.	0.42	1.0	.9979
Tatoosh Is., Washington	1	0.28	0.4	.9156
	2	0.29	0.4	.9430
	3	0.22	0.4	.9857
	4	0.35	0.7	.9674
	5	0.17	0.4	.9905
	6	0.25	0.5	.9898
	7	0.19	0.3	.9889
	8	0.26	0.4	.9806
	9	0.30	0.5	.9749
	10	0.33	0.7	.9689
	11	0.21	0.3	.9558
	12	0.12	0.2	.9638
	yr.	0.25	0.7	.9979

(0.6°C), and the maximum absolute error was 3.6°F (2.0°C). Coastal station combined tests ($n = 192$ station-hour-months) yielded a RMSE of 0.35°F (0.2°C), and a maximum absolute error of 1.0°F (0.6°C).

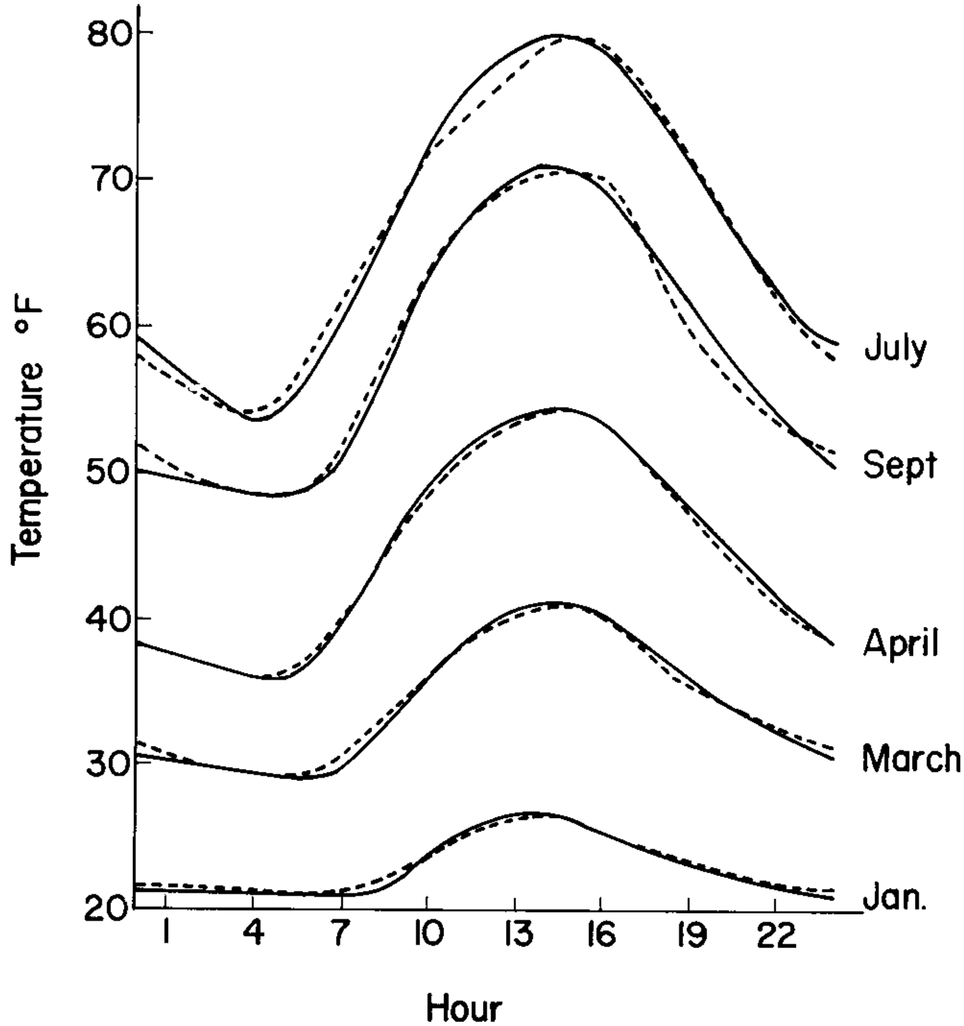

Figure 2. Estimated (solid line) and measured (broken line) mean daily temperature cycle for selected months, Coeur d'Alene, Idaho.

Discussion and Conclusions

The simple model, based on the Pearson type III distribution, can be used to accurately estimate the mean daily temperature cycle, by month, for climatic stations in the Pacific Northwest, provided that the mean daily maximum and minimum temperatures are known. The same monthly coefficients are used for all inland stations, from Butte, Montana to Olympia, Washington—desert, forest, or mountain pass; but a separate set of coefficients is needed for coastal stations influenced by diurnal land-sea breezes and summer morning fog and clouds (Fig. 3).

The region around southern Puget Sound and the lower Columbia River appears to exhibit characteristics of a transition zone, particularly in summer, although stations in these areas exhibit more of the characteristics of interior stations than of coastal stations. Nevertheless, examination of cloud cover data (Pacific Northwest River Basins

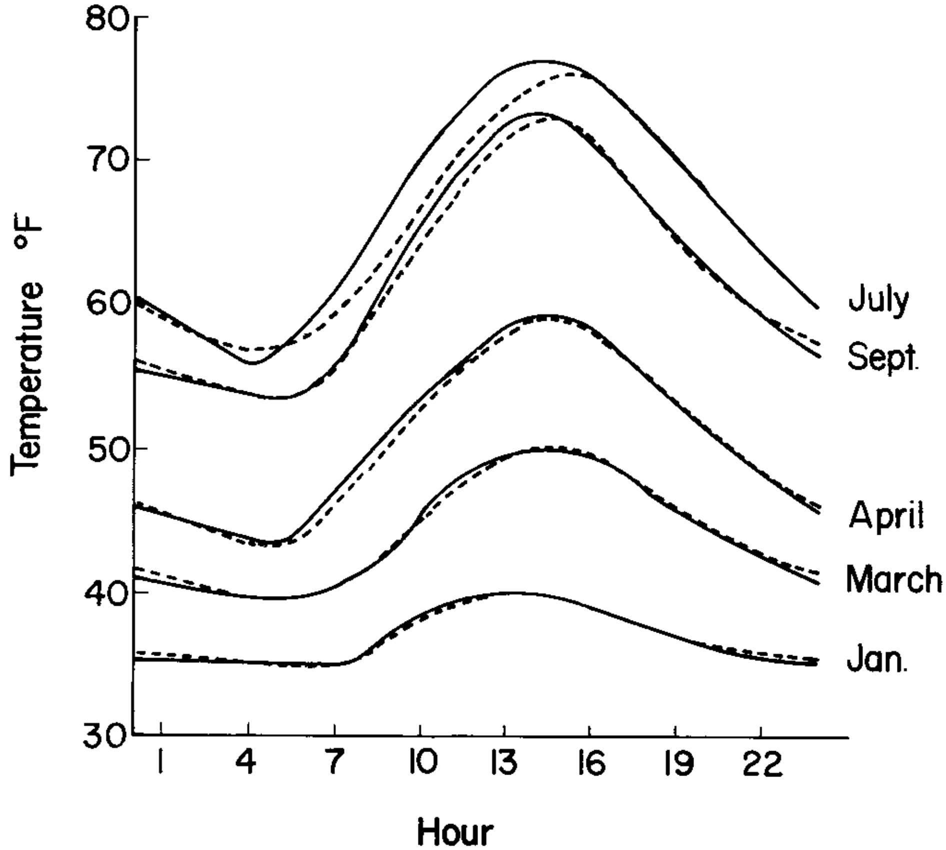

Figure 3. Estimated (solid line) and measured (broken line) mean daily temperature cycle for selected months, Portland, Oregon.

Commission 1968), suggested that the large deviations of estimated from observed temperatures in the morning in summer for Portland, Oregon, were a result of the greater morning cloudiness at that station. Predicted temperatures consistently exceeded observed temperatures, which is what would be expected if greater cloudiness delayed surface heating in the morning. A similar, but less pronounced, effect also occurs at Olympia, Washington.

The model and coefficients presented herein should be applicable for estimating the mean temperature at any given hour, by month, at most stations in the Pacific Northwest. There may be some risk in applying the model to stations located in frost pockets or other areas of poor nocturnal air drainage that might be expected to distort the normal daily temperature pattern.

Coefficients should probably be developed separately if the model is applied outside the region in which it was developed. The coefficients are clearly sensitive to daylength and probably to different diurnal patterns and amounts of cloudiness.

Finally, the model should not be used to predict the temperature pattern for any specific day.

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