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Modulation and Processing Gain Tradeoffs in DS-CDMA Spread Spectrum Systems

David K. Asano†, Tatsuji Hayashi† and Ryuji Kohno‡

†Department of Information Engineering,
Shinshu University, Nagano, Japan

‡Division of Electrical & Computer Engineering,
Yokohama National University, Yokohama, Japan

Abstract — In this paper, Continuous Phase Modulation schemes are examined for use in DS-CDMA spread spectrum systems. Since the power spectra of Continuous Phase Modulation schemes are narrower than BPSK schemes, a larger processing gain can be used for the same overall bandwidth after spreading. The processing gain, relative to BPSK, that can be used with Continuous Phase Modulation is calculated. As an example, it is shown that the performance of an MSK system is better than a BPSK system for an equivalent overall bandwidth.

I. Introduction

Spread Spectrum systems have achieved great popularity and is the subject of intensive research. In most research, more emphasis has been placed on parameters such as the spreading code and receiver structure than on the modulation scheme itself. In direct sequence spread spectrum systems, the modulation scheme that is used is usually two or four level phase shift keying (BPSK or QPSK).

In this paper, we examine Continuous Phase Modulation (CPM) for use in spread spectrum systems and compare the performance with that obtained using BPSK. Our motivation for this work is that a more bandwidth efficient modulation scheme allows the use of a larger processing gain. Although a more bandwidth efficient modulation scheme alone generally results in poorer bit-error-rate performance, we expect that the overall performance can be improved by using a bandwidth efficient modulation scheme and a larger processing gain. We also expect that there is a tradeoff between the overall performance and the unspread bandwidth of the modulation scheme.

II. System Description

In this paper, we consider CPM-type DS-CDMA spread spectrum systems as investigated in [1]. In such systems, each user is given a set of M spreading sequences in order to transmit $\log_2 M$ bits per symbol interval T. For the kth user, these sequences are denoted by $\alpha_1^{(k)}, \cdots, \alpha_M^{(k)}$. Each sequence consists of N chips, so the ith chip of the jth code for user k is denoted by $\alpha_{j,i}^{(k)}$ and is either 1 or -1, i.e., $\alpha_{j,i}^{(k)} \in \{1, -1\}$.

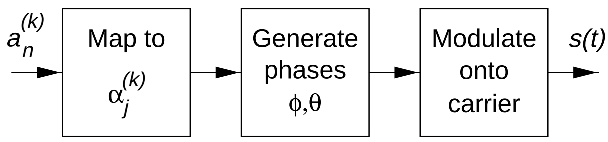

Fig. 1. A spread spectrum CPM signal transmitter.

The process to produce the spread spectrum CPM signal is shown in Fig. 1. First, the input data, $a_n^{(k)}$, is mapped to one of the M spreading sequences, $\alpha_j^{(k)}$.

Second, the chips are mapped to a CPM phase function. For the jth code for user k, this phase function is given by

$$\phi(t, \alpha_j^{(k)}) = 2\pi h \sum_{i=0}^{N-1} \alpha_{j,i}^{(k)} q(t - iT_c), \tag{1}$$

where h is the modulation index and T_c is the chip interval and is equal to T/N. The phase smoothing function, $q(t)$, is a continuous function which has properties given by

$$q(t) = \begin{cases} 0 & t \le 0 \\ 1/2 & t > LT_c \end{cases}, \tag{2}$$

where L is a positive integer which determines the memory in the modulation scheme. For $L = 1$, full response CPM schemes result and for $L \ge 2$, partial response CPM schemes result.

Third, the individual phase functions are combined to produce the final CPM phase function, which is given by

$$\theta(t) = \sum_j \phi(t - jT, \beta_j^{(k)}), \tag{3}$$

where $\beta_j^{(k)} \in \{\alpha_1^{(k)}, \cdots, \alpha_M^{(k)}\}$

Finally, the phase is modulated onto a carrier of frequency f_c to produce the spread spectrum CPM signal, given by

$$s(t) = A\cos[2\pi f_c t + \theta(t) + \theta_0]. \tag{4}$$

In this equation, A is the signal amplitude and θ is an initial phase.

We note here that as a result of spread spectrum modulation, the input data is processed so that the

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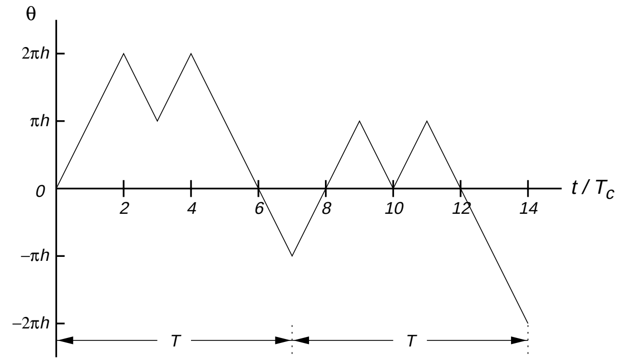

Fig. 2. A typical phase function for a full response spread spectrum CPM signal.

bandwidth of the resulting spread spectrum signal is increased by a factor of G, i.e., the processing gain of the system is G. This also corresponds to the length of the spreading sequence, N, i.e., $G = N$.

In this paper, we consider binary input data, so $a_n^{(k)} \in \{-1, 1\}$. Therefore, $M = 2$ and each user is given two spreading sequences. We also consider full response CPM signals, i.e., $L = 1$.

As an example, we consider Continuous Phase FSK (CPFSK), which has a phase smoothing function defined by

$$q(t) = \begin{cases} 0 & t < 0 \\ \dfrac{t}{2T} & 0 \le t \le T_c \\ 0.5 & t > T_c \end{cases}, \tag{5}$$

which is a straight line for $0 \le t \le T_c$. When $h = 0.5$, Minimum Shift Keying (MSK) results.

Using the CPFSK phase smoothing function, a typical phase function, θ, for a spread spectrum CPM system is shown in Fig. 2. Notice that the phase is continuous, but can increase or decrease at the end of every chip interval.

III. Performance Measures

There are two parameters of interest: bit-error-rate (BER) and bandwidth. The BER and bandwidth are closely related. In general, a larger transmitted signal bandwidth leads to better BER performance if the transmitted signal is designed properly. However, a larger bandwidth means that the processing gain that can be used is lower since the overall bandwidth after spreading is fixed by spectrum limits. Therefore, we first look at the bandwidth properties of unspread signals and determine the processing gains that are possible. After that, we consider the BER performance that can be obtained with the processing gain that can be used.

A. Bandwidth

There are many ways of defining bandwidth. In this paper, we use the fractional power containment band-

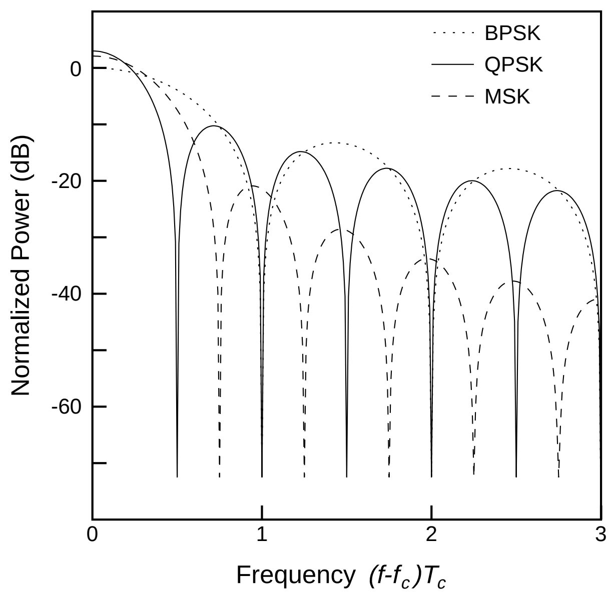

Fig. 3. Power spectral densities of QPSK and MSK compared to BPSK.

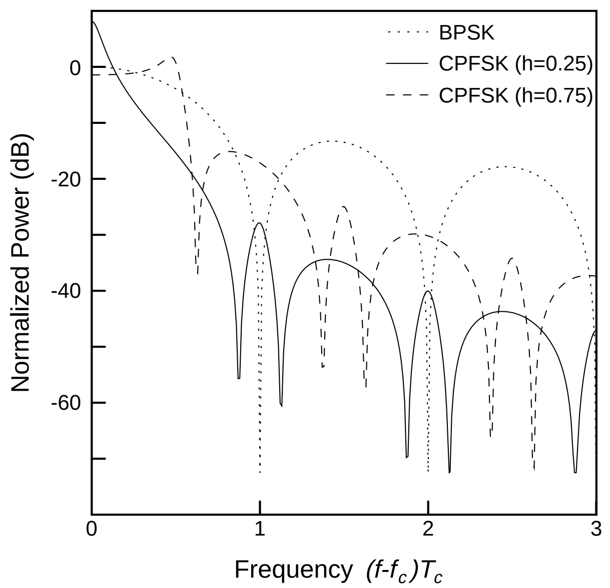

Fig. 4. Power spectral densities of CPFSK compared to BPSK.

width, denoted by $B_{x\%}$, which is the bandwidth that contains $x\%$ of the signal power. For example, the 99% bandwidth, $B_{99\%}$, is the bandwidth which contains 99% of the signal power.

For comparison, the power spectral density (PSD) of BPSK is compared with those of QPSK and MSK in Fig. 3 and with that of CPFSK in Fig. 4. These PSDs were determined using the method described in [2] assuming that random spreading sequences are used.

To get a better feel for the bandwidth comparisons, the percentage of the total power contained as a function of the frequency offset from the center frequency

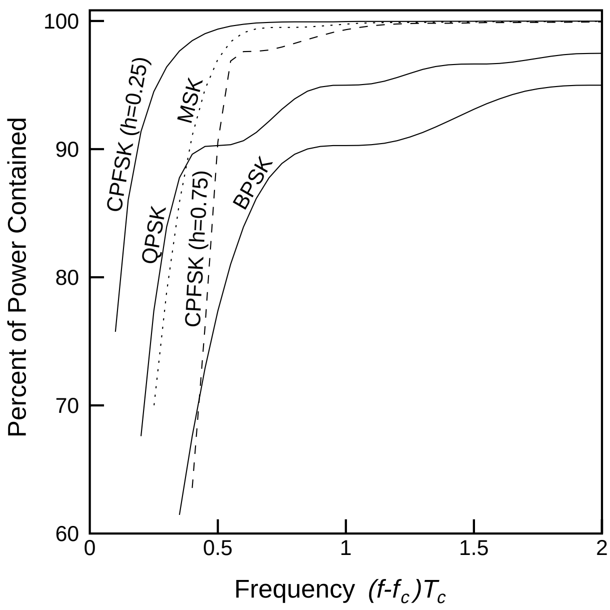

Fig. 5. The power containment properties of the modulation schemes.

Table I
Power Containment Bandwidths

Modulation	$B_{90\%}T_c$	$B_{99\%}T_c$
BPSK	0.849	10.29
QPSK	0.424	5.156
CPFSK ($h = 0.25$)	0.184	0.449
MSK ($h = 0.5$)	0.388	0.591
CPFSK ($h = 0.75$)	0.498	0.928

is shown in Fig. 5. From this figure, we can see that BPSK and QPSK show a slower convergence to 100% power containment than CPFSK schemes. Therefore, CPFSK schemes are more bandwidth efficient.

Specifically, the 90% and 99% power containment bandwidths of CPFSK schemes, normalized by the chip period T_c, are shown as a function of h in Fig. 6. This figure shows the gradual increase in the power containment bandwidth as h increases.

Comparing the bandwidth values for CPFSK to BPSK and QPSK in Table I, we see that the bandwidth requirements for CPFSK are much lower. Therefore, a larger processing gain can be used in conjunction with CPFSK schemes. Since we are interested in keeping the total bandwidth of the system after spreading constant for all modulation schemes, i.e.,

$$BG = constant, \tag{6}$$

where B is the bandwidth before spreading.

B. Bit-Error-Rate

In general, the BER of a spread spectrum system is difficult to calculate. The BER depends not only on

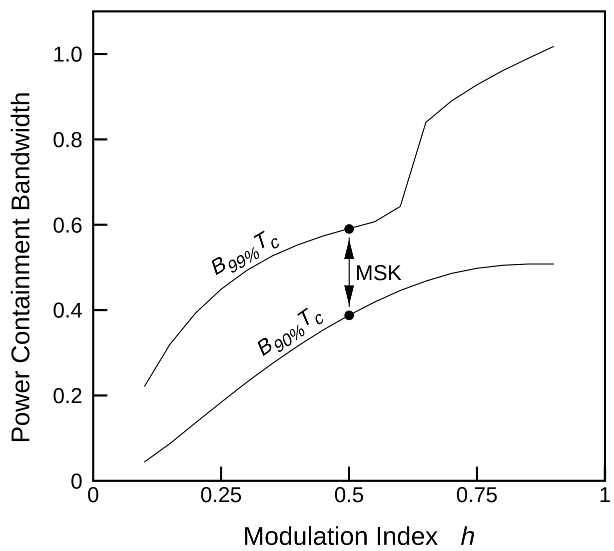

Fig. 6. The 90% and 99% power containment bandwidths for CPFSK as a function of h.

the modulation scheme and spreading sequences used, but also on the multiple access interference due to the presence of other users and also on the ever present thermal noise, which is generally modelled as Additive White Gaussian Noise (AWGN).

For unspread modulation schemes, the BER performance can be expressed in terms of a correlation function, ρ, as [3]

$$P_e = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{(1-\rho)SNR}{2}}\right]. \tag{7}$$

The best performance is obtained when $\rho = -1$. BPSK schemes have a correlation equal to -1 and are "optimal" in this sense.

For CPFSK modulation schemes, ρ is given by

$$\rho = \frac{\sin(2\pi h)}{2\pi h}, \tag{8}$$

which is plotted in Fig. 7. Notice that the performance improves as h is increased up until $h \approx 0.7$, after which the performance worsens.

Of course, this discussion is only for unspread modulation schemes. To determine the performance for DS-CDMA schemes, the effects of multiple access interference must be included. However, this gives some indication of performance that can be expected.

To get some indication of the BER performance that can be obtained for DS-CDMA systems, we examine received signal-to-noise ratio (SNR), which includes the effects of multiple access interference and AWGN. For BPSK, the received SNR is given by [4]

$$SNR = \left[\frac{K-1}{3G} + \frac{N_0}{2E_b}\right]^{-1/2}, \tag{9}$$

where E_b/N_0 is the signal-to-noise ratio of the desired

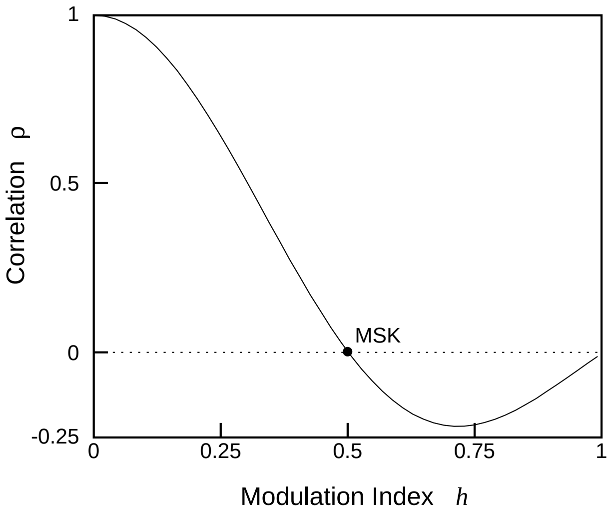

Fig. 7. The correlation function ρ for CPFSK signals.

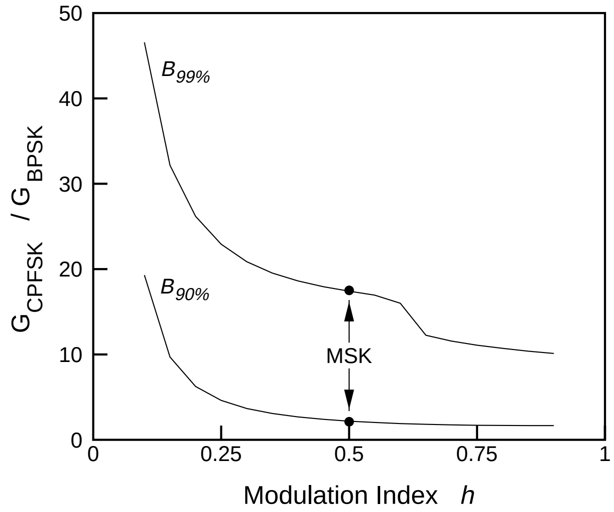

Fig. 8. The processing gain of a CPFSK system relative to a BPSK system as a function of h.

signal alone and K is the number of users. The corresponding SNR for MSK is given by [5]

$$SNR = \left[\left(\frac{15 + 2\pi^2}{12\pi^2} \right) \frac{K-1}{G} + \frac{N_0}{2E_b} \right]^{-1/2}. \quad (10)$$

IV. Performance Comparisons

Using (6) allows us to express the processing gain of a CPFSK system as a function of the processing gain of the BPSK system as

$$G_{CPFSK} = \frac{B_{BPSK}}{B_{CPFSK}} G_{BPSK} \quad (11)$$

This relationship is plotted in Fig. 8 as a function of the modulation index h. We see that even for large modulation indices, the processing gain for CPFSK can be made roughly 1.6 times and 10 times larger than a BPSK system for the same 90% and 99% containment bandwidths, respectively.

Combining this result with Fig. 7 shows that there should be some tradeoff between the unspread signal bandwidth, controlled by the modulation index h, and the performance achievable after spreading. As h increases, the bandwidth increases, but the BER decreases. Therefore, what is the best combination of modulation and processing gain? We hope to answer this question in future work.

As a specific example, we compare an MSK system to a BPSK system. Using a processing gain of 127 for BPSK and given the constraint of constant bandwidth after spreading, the processing gains that can be used for an MSK system are given in Table II. The processing gain that can be used also depends on the bandwidth of interest, so two values for MSK are given.

Using these values for the processing gain in (9) and (10), the SNR values are plotted for BPSK and MSK in Fig. 9 for $K = 7$ users. We can see that MSK offers

Table II
Processing Gains for Equal Bandwidths after Spreading

Bandwidth	G_{BPSK}	G_{MSK}
$B_{90\%}$	127	277
$B_{99\%}$	127	2211

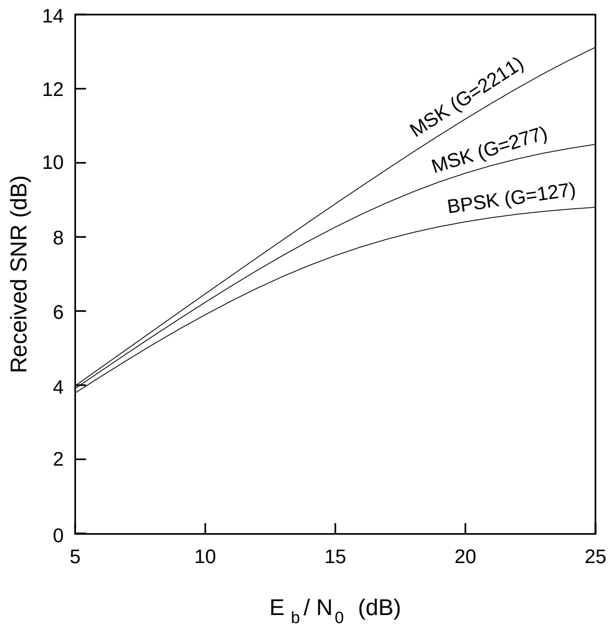

Fig. 9. Received SNR for $G_{BPSK} = 127$.

a larger received SNR than BPSK and hence better performance for the same spread bandwidth.

V. Conclusions

In this paper, Continuous Phase Modulation (CPM) was considered for use in DS-CDMA systems. The spectral efficiency of CPM allows for the use of a larger processing gain than BPSK for the same bandwidth after spectrum spreading.

Specifically, Continuous Phase FSK (CPFSK) was considered and compared with Binary Phase Shift Keying (BPSK). The processing gain that can be used for CPFSK was found to be 1.6 to 40 times the processing gain of BPSK, depending on the bandwidth measure and modulation index used. As an example, the received SNR for MSK and BPSK systems was compared, under the constraint of constant bandwidth after spreading, and it was found that better performance could be obtained by using CPM.

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