Comparision of ODE Solver Results with Pejsa's Software

Generate Interpolation of F Function

Velocity *Distance* *Time* *Acceleration* *F Function*

$D :=$

	0	1	2	3	4
0	500	$1.995 \cdot 10^4$	19.962	16.98	$1.472 \cdot 10^4$
1	550	$1.854 \cdot 10^4$	17.285	20.55	$1.472 \cdot 10^4$
2	600	$1.726 \cdot 10^4$	15.054	24.45	...

$$F0(v, BC) := BC \cdot interp\left(cspline\left(D^{\langle 0 \rangle} \frac{ft}{s}, D^{\langle 4 \rangle} \cdot ft\right), D^{\langle 0 \rangle} \cdot \frac{ft}{s}, D^{\langle 4 \rangle} ft, v\right)$$

Example Projectile/Rifle Characteristics

$BC := 0.4$

$v_0 := 2400$

$S := 1.5$

$Z := 139$

Setup Drag Models

$$n1(v) := \begin{cases} 0.5 & \text{if } v > 1400 \\ 0.000001 & \text{if } 1200 < v \le 14 \\ -3 & \text{if } 900 < v \le 1200 \\ 0.000001 & \text{otherwise} \end{cases} \qquad a(v) := \begin{cases} 166 & \text{if } v > 1400 \\ 6211 & \text{if } 1200 < v \le 1400 \\ 1.0733 \cdot 10^{13} & \text{if } 900 < v \le 1200 \\ 14723 & \text{otherwise} \end{cases}$$

$$A(v, BC) := \frac{v^{1-n1(v)}}{a(v) \cdot BC}$$

Setup ODE System and Solve for Projectile X and Y Velocity

$N := 4000 \qquad T := 1.8$

Given

$$\frac{d}{dt} v(t) = -\frac{v(t)^{2-n1(v(t))}}{a(v(t)) \cdot BC} \qquad v(0) = v_0$$

$$\frac{d}{dt} vy(t) = -\frac{v(t)^{1-n1(v(t))}}{a(v(t)) \cdot BC} \cdot vy(t) + \frac{g}{\frac{ft}{s^2}} \qquad vy(0) = 0$$

$$\begin{pmatrix} f \\ h \end{pmatrix} := Odesolve\left[\begin{pmatrix} v \\ vy \end{pmatrix}, t, T, N\right]$$

Gather Data on Comparable Projectile From Pejsa's Appendix

Range *Velocity* *Height*

$$\mathbf{Data_P} := \begin{pmatrix} 0 & 2400 & -1.5 \\ 50 & 2293 & 0.54 \\ 100 & 2188 & 0.93 \\ 150 & 2085 & -0.49 \\ 200 & 1985 & -3.92 \\ 250 & 1888 & -9.56 \\ 300 & 1792 & -17.6 \\ 350 & 1700 & -28.4 \\ 400 & 1610 & -42.2 \\ 450 & 1522 & -59.3 \\ 500 & 1437 & -80.2 \end{pmatrix}$$

Integrate olver Velocity Results to Obtain X and Y Position Data

$\mathbf{i} := 0 .. 180$ $\quad \mathbf{t1} := 0 , 0.1 , 1.8$ $\quad \mathbf{Time_i} := \mathbf{i} \cdot 0.01$ $\quad \mathbf{V} := \overrightarrow{\mathbf{f(Time)}}$

$\mathbf{vel(t)} := \mathbf{interp(cspline(Time , V) , Time , V , t)}$

$$\mathbf{D3} := \left| \begin{array}{l} \textbf{for} \quad \mathbf{i} \in 0 .. 180 \\ \quad \delta_{\mathbf{i}} \leftarrow \displaystyle\int_0^{\mathbf{i} \cdot 0.01} \mathbf{h(t)} \ \mathbf{dt} \\ \delta \end{array} \right. \qquad \mathbf{D4} := \left| \begin{array}{l} \textbf{for} \quad \mathbf{i} \in 0 .. 180 \\ \quad \delta_{\mathbf{i}} \leftarrow \displaystyle\int_0^{\mathbf{i} \cdot 0.01} \mathbf{f(t)} \ \mathbf{dt} \\ \delta \end{array} \right.$$

$\mathbf{drop(t)} := \mathbf{interp(cspline(Time , D3) , Time , D3 , t)}$

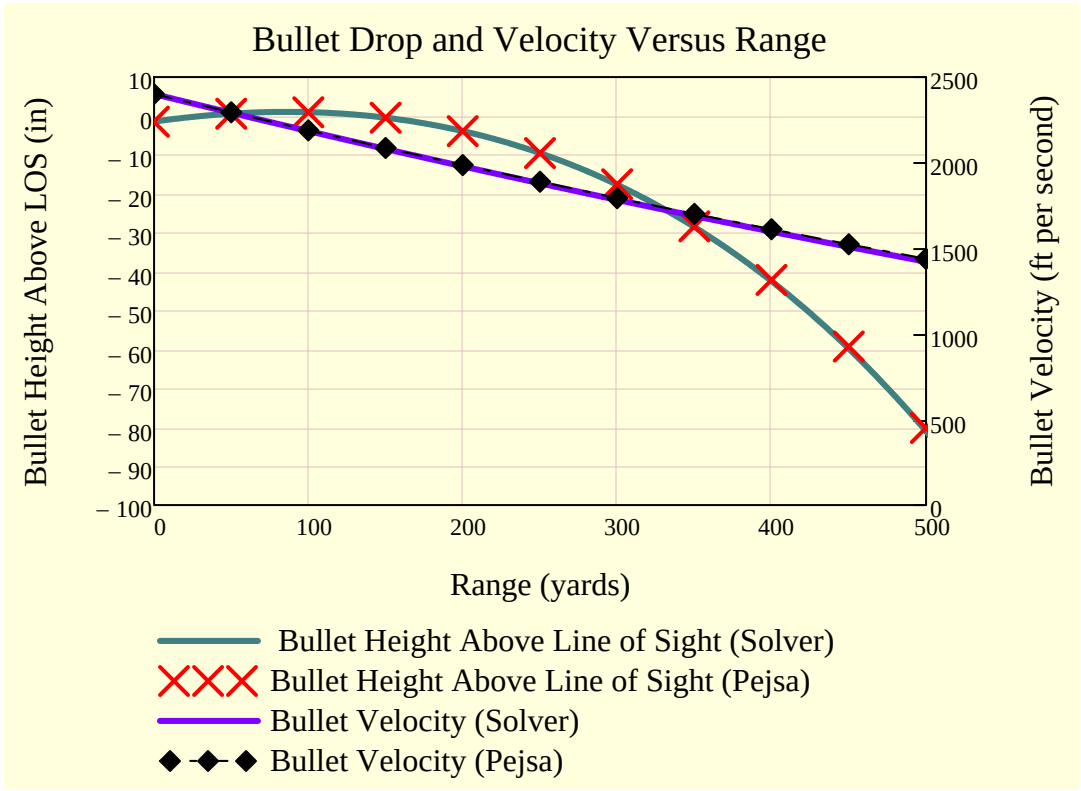
$\mathbf{dist(t)} := \mathbf{interp(cspline(Time , D4) , Time , D4 , t)}$

$\mathbf{t} := 0 , 0.001 .. 1.8$

$\mathbf{t*(d)} := \zeta \leftarrow \mathbf{root(dist(b) - 3d , b , 0 , 1.21)}$

$\mathbf{drp(d)} := 12 \cdot \mathbf{drop(t*(d))}$

$$\mathbf{H}(\zeta , \mathbf{S} , \delta , \mathbf{Z}) := -(\zeta + \mathbf{S}) + (\mathbf{drp(Z)} + \mathbf{S}) \cdot \frac{\delta}{\mathbf{Z}}$$


Bullet Drop and Velocity Versus Range
Bullet Height Above LOS (in)
10
0
– 10
– 20
– 30
– 40
– 50
– 60
– 70
– 80
– 90
– 100
0
100
200
300
400
500
2500
2000
1500
1000
500
0
Bullet Velocity (ft per second)
Range (yards)
Bullet Height Above Line of Sight (Solver)
Bullet Height Above Line of Sight (Pejsa)
Bullet Velocity (Solver)
Bullet Velocity (Pejsa)