Generate Interpolation of F Function

$\mathbf{D} :=$

	Velocity 0	Distance 1	Time 2	Acceleration 3	F Function 4
0	500	$1.995\cdot10^4$	19.962	16.98	$1.472\cdot10^4$
1	550	$1.854\cdot10^4$	17.285	20.55	$1.472\cdot10^4$
2	600	$1.726\cdot10^4$	15.054	24.45	...

$$\mathbf{F0}(\mathbf{v},\mathbf{BC}) := \mathbf{BC}\cdot \mathbf{interp}\left(\mathbf{cspline}\left(\mathbf{D}^{\langle 0\rangle}\frac{\mathbf{ft}}{\mathbf{s}},\mathbf{D}^{\langle 4\rangle}\cdot\mathbf{ft}\right),\mathbf{D}^{\langle 0\rangle}\cdot\frac{\mathbf{ft}}{\mathbf{s}},\mathbf{D}^{\langle 4\rangle}\,\mathbf{ft},\mathbf{v}\right)$$

Pejsa's work strongly depends on the data from the standard projectile. This table is in all of my analysis work.

Determine Critical Ranges

$$\mathbf{R_{Critical}}(\mathbf{V_0},\mathbf{V_F},\mathbf{n},\mathbf{BC}) := \mathbf{root}\left[\mathbf{V_0}\cdot\left(1-\frac{3\cdot\mathbf{n}\dfrac{\mathbf{R}}{\mathbf{yd}}}{\dfrac{\mathbf{F0}(\mathbf{V_0},\mathbf{BC})}{\mathbf{ft}}}\right)^{\frac{1}{\mathbf{n}}}-\mathbf{V_F},\mathbf{R},0\mathbf{yd},1000\mathbf{yd}\right]$$

This is a simple root finder that works for this example – it will not work for every case. I could come up with a general solution, but this is sufficient for what I am doing in this example.

$\mathbf{V_0} := 2900\frac{\mathbf{ft}}{\mathbf{s}}$ $\mathbf{BC} := 0.4$ $\mathbf{S} := 1.5\mathbf{in}$ $\mathbf{H_m} := 1\mathbf{in}$

$$\mathbf{CritVel} := \left(1400\frac{\mathbf{ft}}{\mathbf{s}}\quad 1200\frac{\mathbf{ft}}{\mathbf{s}}\quad 900\frac{\mathbf{ft}}{\mathbf{s}}\right)^{\mathbf{T}}$$

Velocities at which exponents change.

$$\mathbf{n1} := \left(0.5\quad 1\times10^{-6}\quad -3\right)^{\mathbf{T}}$$

Pejsa model exponents. Because zero causes issues and would have to be handled as a special case, I will just assume a very small number. It will make no difference in my solution.

$$\mathbf{R_1} := \overrightarrow{\mathbf{R_{Critical}}\left[\begin{pmatrix}\mathbf{V_0}\\ \mathbf{CritVel}_0\\ \mathbf{CritVel}_1\end{pmatrix},\mathbf{CritVel},\mathbf{n1},\mathbf{BC}\right]}$$

$$\mathbf{R_{Breakpoint}} := \left|\begin{array}{l}\zeta \leftarrow \begin{pmatrix}0\\0\\0\end{pmatrix}\\ \mathbf{for}\ \ \mathbf{i}\in 0..\,2\\ \quad \mathbf{for}\ \ \mathbf{j}\in 0..\,\mathbf{i}\\ \quad\quad \zeta_{\mathbf{i}} \leftarrow \mathbf{R_1}_{\mathbf{j}} + \zeta_{\mathbf{i}}\\ \zeta\end{array}\right. = \begin{pmatrix}727.5\\855.2\\1233.5\end{pmatrix}\cdot\mathbf{yd}$$

These are the ranges at which the bullet's velocity transitions from one velocity region to another.

$$F_m(v_0, BC, n, R) := F0(v_0, BC) - \left(0.75 + 0.00006 \cdot \frac{R}{yd}\right) \cdot n \cdot \frac{R}{yd} \cdot ft$$

$$D(n, v_0, BC, R) := \left| \begin{array}{l} F1 \leftarrow F_m(v_0, BC, n, R) \\ \left[\dfrac{\dfrac{41.68}{\left(\dfrac{v_0}{\frac{ft}{s}}\right)}}{\dfrac{1}{\frac{R}{yd}} - \dfrac{1}{\frac{F1}{ft}}} \right]^2 in \end{array} \right.$$

This is the formula Pejsa lists in his Appendix. He uses different formulas for Fm throughout the book.

- $F0 - 0.75 \cdot n \cdot R$
- $F0 - 0.78 \cdot n \cdot R$
- $F0 - 0.80 \cdot n \cdot R$
- $F0 - (0.75 + 0.00006\, R) \cdot n \cdot R$

I found this very irritating as I tried to duplicate his results.

$$v_{Critical}(n, v_0, BC, R1) := \frac{2 \dfrac{D(n, v_0, BC, R1)}{in}}{\dfrac{R1}{yd}} \cdot \left[\frac{1 + 0.78 \cdot n \cdot \left(\dfrac{\frac{R1}{yd}}{\frac{F_m(v_0, BC, n, R1)}{ft}} \right)^2}{1 - \dfrac{\frac{R1}{yd}}{\frac{F_m(v_0, BC, n, R1)}{ft}}} \right] \cdot \frac{in}{yd}$$

Pejsa used this equation in his text examples.The coefficient 0.78 is one that he varied a bit.

$$v_C := \overrightarrow{v_{Critical}\left[n1, \begin{pmatrix} V_0 \\ CritVel_0 \\ CritVel_1 \end{pmatrix}, BC, R_1 \right]} = \begin{pmatrix} 0.649 \\ 0.265 \\ 1.267 \end{pmatrix} \cdot \frac{in}{yd}$$

These are the downward velocities of the projectile at the end of each of the critical velocity regions.

$$D_R(v_0, BC, R1) := \left| \begin{array}{l} \text{"Drop Formula"} \\ d1 \leftarrow D(n1_0, v_0, BC, \min(R1, R_{Breakpoint_0})) \\ d2 \leftarrow D\left(n1_1, 1400\frac{ft}{s}, BC, \min(R1 - R_{Breakpoint_0}, R_{Breakpoint_1} - R_{Breakpoint_0})\right) \text{ if } R1 > R_{Breakpoint_0} \\ dv2 \leftarrow v_{C_0} \cdot (R1 - R_{Breakpoint_0}) \text{ if } R1 > R_{Breakpoint_0} \\ d3 \leftarrow D\left(n1_2, 1200\frac{ft}{s}, BC, \min(R1 - R_{Breakpoint_1}, R_{Breakpoint_2} - R_{Breakpoint_1})\right) \text{ if } R1 > R_{Breakpoint} \\ dv3 \leftarrow v_{C_1} \cdot (R1 - R_{Breakpoint_1}) \text{ if } R1 > R_{Breakpoint_1} \\ d4 \leftarrow D\left(0, 900\frac{ft}{s}, BC, \min(R1 - R_{Breakpoint_2}, 10000yd - R_{Breakpoint_2})\right) \text{ if } R1 > R_{Breakpoint_2} \\ dv4 \leftarrow v_{C_2} \cdot (R1 - R_{Breakpoint_2}) \text{ if } R1 > R_{Breakpoint_2} \\ d1 + d2 + d3 + d4 + dv2 + dv3 + dv4 \end{array} \right.$$

$$H(R, Z, S, v_0, BC) := -(D_R(v_0, BC, R) + S) + \frac{(D_R(v_0, BC, Z) + S) \cdot R}{Z}$$

$$M(H_m, S, V_0, BC) := \begin{array}{|l} \text{"Implementation of Pejsa's Midpoint Formula"} \\ F \leftarrow F0(V_0, BC) \\ G \leftarrow 41.68 \\ \dfrac{1}{\dfrac{G}{\left(\dfrac{V_0}{\frac{ft}{s}} \cdot \sqrt{\dfrac{H_m}{in} + \dfrac{S}{in}}\right)} + \dfrac{2}{\frac{F}{ft}}} \cdot yd \end{array}$$

Range
Velocity
Height

$$Data_P := \begin{pmatrix} 0 & 2900 & -1.5 \\ 100 & 2662 & 1.70 \\ 200 & 2434 & 0 \\ 300 & 2216 & -7.6 \\ 400 & 2008 & -22.3 \\ 1000 & 1043 & -386.7 \end{pmatrix}$$

From Pejsa page 91.

$M_{Range} := M(1in, 1.5in, V_0, BC) = 103.635 \cdot yd$ ✓

$$SH := \sqrt{1 + \frac{\frac{S}{in}}{\frac{H_m}{in}}}$$

$Z := 200yd$

$i := 0 .. 10$

$R_i := 0.00001yd + 100 \cdot yd \cdot i$

A zero value will cause an error. Adding a small increment ensures I do not encounter a zero.

$R =$ (· yd)

	0
0	$1 \cdot 10^{-5}$
1	100
2	200
3	300
4	400
5	500
6	600
7	700
8	800
9	900
10	$1 \cdot 10^{3}$

$\zeta := \overrightarrow{H(R, Z, S, V_0, BC)} =$ (· in) ✓

	0
0	-1.5
1	1.0
2	-1.4
3	-9.7
4	-25.1
5	-49.2
6	-83.9
7	-131.8
8	-196.3
9	-282.0
10	-394.3