Reference Index

- Wikipedia reference on dynamical parallax — *Link*
- Amazon link to book *Parallax* — *Link*
- Wikipedia article on mass-luminosity relation — *Link*
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- Types of binary star systems — *Link*
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Analysis

Units

$ly := c \cdot yr$ — Definition of light-year

$AU := 93 \cdot 10^{6} \cdot mi$ — Astronomical Unit

$L_{\odot} := 3.846 \cdot 10^{26} W$ — Luminosity of the Sun

$arcsec := \frac{deg}{60 \cdot 60}$ — Definition of arcsec

$pc := \frac{AU}{arcsec} = 3.263 \cdot ly$ — Parsec

$m_{\odot} := 1.989 \cdot 10^{30} \cdot kg$ — Mass of the Sun

$mas := 0.001 \cdot \frac{deg}{60 \cdot 60}$ — Milli-arcsec defintion

Constants

$G_U := 6.6742 \cdot 10^{-11} \cdot \frac{m^3}{kg \cdot sec^2}$ — Universal Gravitational Constant

$M_{Sun} := -26.74$ — Absolute magnitude of the Sun

$d_{Sun} := 1AU$ — Distance of the Sun

Orbital Radius Based on Revolution Period *Link*

$$G_U \cdot \frac{(m_1 + m_2) \cdot m_2}{R^2} = m_2 \cdot \omega^2 \cdot R \Rightarrow R^3 = \frac{G_U \cdot (m_1 + m_2)}{\omega^2} = \frac{G_U \cdot (m_1 + m_2) \cdot T^2}{4 \cdot \pi^2}$$

$$R(T, M_T) := \frac{4^{\frac{2}{3}} \cdot G_U^{\frac{1}{3}} \cdot M_T^{\frac{1}{3}} \cdot T^{\frac{2}{3}}}{4 \cdot \pi^{\frac{2}{3}}}$$

We can use this formula to compute the semi-major axis of the orbit knowing only the mass and period.

Luminosity-Mass Relationship

Link

$$L_{Empirical}(m_{Star}) := \text{"Mass Luminosity Relationship"}$$

$$0.23 \cdot \left(\frac{m_{Star}}{m_\odot}\right)^{2.3} \quad \text{if } m_{Star} < 0.43 \cdot m_\odot$$

$$\left[\left(\frac{m_{Star}}{m_\odot}\right)^{4}\right] \quad \text{if } 0.43 \cdot m_\odot \le m_{Star} < 2 \cdot m_\odot$$

$$1.5\left(\frac{m_{Star}}{m_\odot}\right)^{3.5} \quad \text{if } 2 \cdot m_\odot \le m_{Star} < 21.459692159995348191 \cdot m_\odot$$

$$3200 \cdot \frac{m_{Star}}{m_\odot} \quad \text{if } m_{Star} \ge 21.459692159995348191 \cdot m_\odot$$

$m_{Star} := 6 \cdot m_\odot$ **Starter value**

$$f(L_{Star}) := \text{root}\left(L_{Empirical}(m_{Star}) - \frac{L_{Star}}{L_\odot}, m_{Star}, 0 \cdot m_\odot, 8 \cdot m_\odot\right)$$

Luminosity vs Magnitude Relationship

$$L_{Star}(M_{Star}, d_{Star}) := \frac{10^{\frac{M_{Sun} - M_{Star}}{2.5}} \cdot L_\odot \cdot d_{Star}^{2}}{AU^2}$$

Dynamical Parallax Algorithm

$\lambda(a) :=$ "Case of Separately Resolvable Binary Stars"

$d \leftarrow 2\,AU$

$P \leftarrow a_0 \cdot yr$

$\theta \leftarrow a_1 \cdot arcsec$

$m_1 \leftarrow a_2$

$m_2 \leftarrow a_3$

for $i \in 0 .. 50$

$m_1 \leftarrow f\left(L_{Star}\left(m_1, d\right)\right)$

$m_2 \leftarrow f\left(L_{Star}\left(m_2, d\right)\right)$

$r \leftarrow R\left(P, m_1 + m_2\right)$

$d \leftarrow \frac{r}{\theta}$

$$\left(\frac{d}{ly} \quad \frac{m_1}{m_\odot} \quad \frac{m_2}{m_\odot} \quad \frac{r}{AU}\right)^T$$

Test Cases

Alpha Centuri *Link*

Period (yr)	79.92	$\lambda =$	4.273	**light-year**	**Wikipedia Value**
Orbit Angle (arcsec)	17.57		1.098	**mass of primary**	4.37
Primary Mag (mag)	0.01		0.81	**mass of secondary**	1.1
Sec Mag (mag)	1.33		23.007	**radius in AU**	0.907
					$\frac{11.2 + 35.6}{2}$

$= \begin{pmatrix} 4.37 \\ 1.1 \\ 0.907 \\ 23.4 \end{pmatrix}$

Procyon *Link*

Period (yr)	40.82	$\lambda =$	10.688	**light-year**	**Wikipedia Value**
Orbit Angle (arcsec)	4.3		1.61	**mass of primary**	11.46
Primary Mag (mag)	0.34		0.068	**mass of secondary**	1.499
Sec Mag (mag)	10.7		14.084	**radius in AU**	0.602
					15

70 Ophiuchi *Link*

Period (yr)	88.3	$\lambda =$	15.664	**light-year**	**Wikipedia Value**
Orbit Angle (arcsec)	4.556		0.801	**mass of primary**	16.6
Primary Mag (mag)	4.2		0.542	**mass of secondary**	0.9
Sec Mag (mag)	5.9		21.87	**radius in AU**	0.7
					23.2

Eta Cassiopeiae *Link*

Period (yr)	480	$\lambda =$	18.873	**light-year**	**Wikipedia Value**
Orbit Angle (arcsec)	11.9939		1.048	**mass of primary**	19.42
Primary Mag (mag)	3.44		0.403	**mass of secondary**	1.0386
Sec Mag (mag)	7.51		69.37	**radius in AU**	0.57
					71