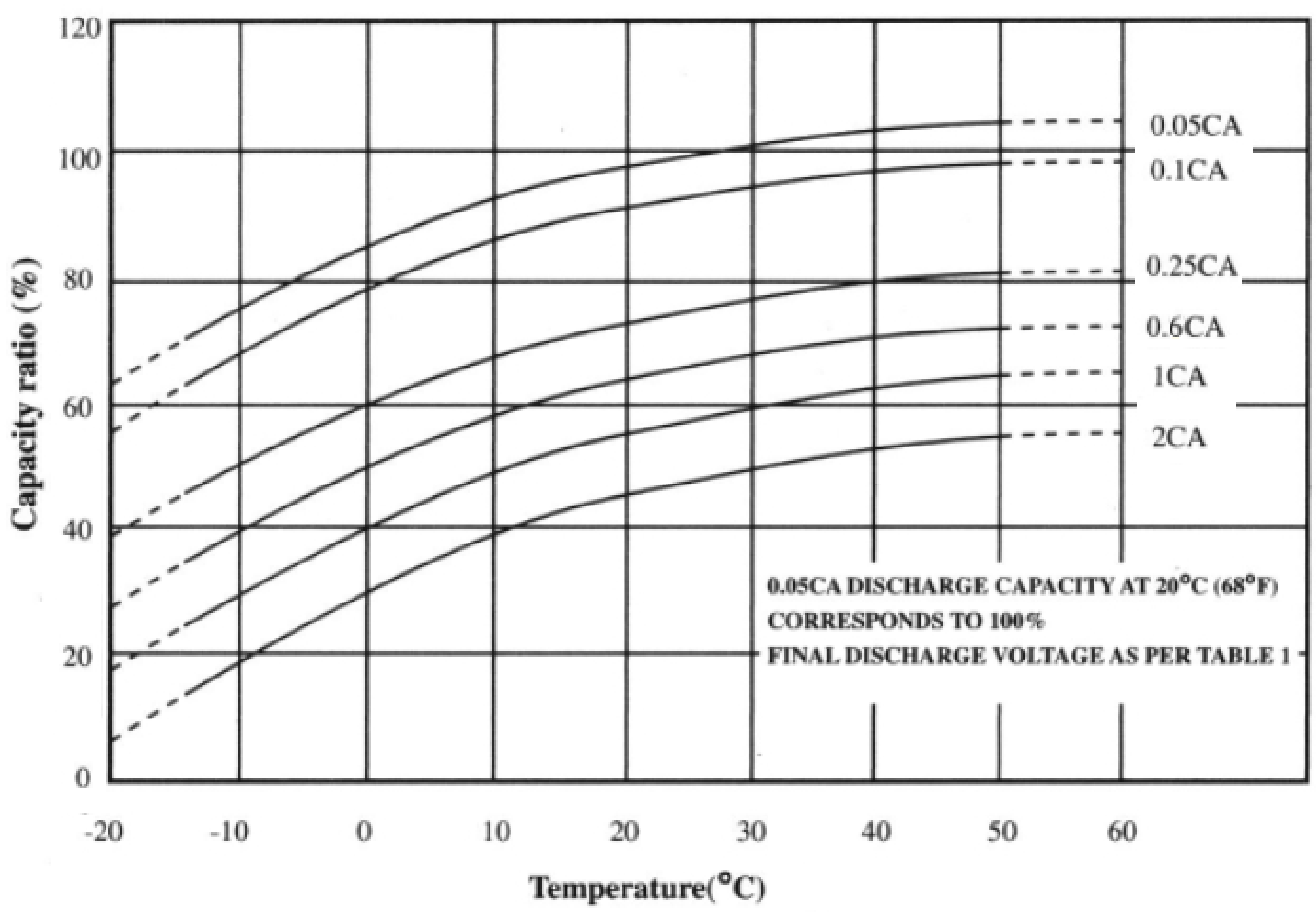
# TECHNICAL NOTE, BATTERY CAPACITY VERSUS DISCHARGE RATE AND TEMPERATURE

## Introduction

I was given the following discharge curves from Innovative Battery Technology and I needed to come up with a simple Excel formula that did not use a macro. I decided to use a polynomial fit to the curves and to implement a polynomial evaluation using Excel's sumproduct function.

## Battery Data **From Innovative Battery Technology**




## Data Capture

**I use Dagra to capture the six curves.**

**Cap05** :=

| | 0 | 1 |
|---|---|---|
| 0 | -20.023 | 0.628 |
| 1 | -19.984 | ... |

**Cap10** :=

| | 0 | 1 |
|---|---|---|
| 0 | -20.007 | 0.551 |
| 1 | -19.974 | ... |

**Cap25** :=

| | 0 | 1 |
|---|---|---|
| 0 | -19.973 | 0.388 |
| 1 | -19.945 | ... |

**Cap60** :=

| | 0 | 1 |
|---|---|---|
| 0 | -20.16 | 0.275 |
| 1 | -20.121 | ... |

**Cap100** :=

| | 0 | 1 |
|---|---|---|
| 0 | -20.034 | 0.173 |
| 1 | -19.989 | ... |

**Cap200** :=

| | 0 | 1 |
|---|---|---|
| 0 | -20.116 | 0.063 |
| 1 | -20.072 | ... |

## Interpolation

$$\mathbf{f}(\mathbf{x}) := \begin{pmatrix} \mathrm{interp}\left(\mathrm{cspline}\left(Cap05^{\langle 0\rangle}, Cap05^{\langle 1\rangle}\right), Cap05^{\langle 0\rangle}, Cap05^{\langle 1\rangle}, x\right) \\ \mathrm{interp}\left(\mathrm{cspline}\left(Cap10^{\langle 0\rangle}, Cap10^{\langle 1\rangle}\right), Cap10^{\langle 0\rangle}, Cap10^{\langle 1\rangle}, x\right) \\ \mathrm{interp}\left(\mathrm{cspline}\left(Cap25^{\langle 0\rangle}, Cap25^{\langle 1\rangle}\right), Cap25^{\langle 0\rangle}, Cap25^{\langle 1\rangle}, x\right) \\ \mathrm{interp}\left(\mathrm{cspline}\left(Cap60^{\langle 0\rangle}, Cap60^{\langle 1\rangle}\right), Cap60^{\langle 0\rangle}, Cap60^{\langle 1\rangle}, x\right) \\ \mathrm{interp}\left(\mathrm{cspline}\left(Cap100^{\langle 0\rangle}, Cap100^{\langle 1\rangle}\right), Cap100^{\langle 0\rangle}, Cap100^{\langle 1\rangle}, x\right) \\ \mathrm{interp}\left(\mathrm{cspline}\left(Cap200^{\langle 0\rangle}, Cap200^{\langle 1\rangle}\right), Cap200^{\langle 0\rangle}, Cap200^{\langle 1\rangle}, x\right) \end{pmatrix}$$

$$\mathbf{h}(\mathbf{t}, \mathbf{x}) := \mathrm{interp}\left[\mathrm{cspline}\left[\begin{pmatrix} 0.05 \\ 0.10 \\ 0.25 \\ 0.60 \\ 1.00 \\ 2.00 \end{pmatrix}, f(x)\right], \begin{pmatrix} 0.05 \\ 0.10 \\ 0.25 \\ 0.60 \\ 1.00 \\ 2.00 \end{pmatrix}, f(x), t\right]$$

## Regression Setup

$$\mathbf{Temp} := \left| \begin{array}{l} \delta \leftarrow (-20\ \ -10\ \ 0\ \ 10\ \ 20\ \ 30\ \ 40\ \ 50\ \ 60)^T \\ \textbf{for}\ \ i \in 0..5 \\ \quad \alpha \leftarrow \mathbf{stack}(\delta, \alpha) \\ \mathbf{submatrix}(\alpha, 0, \mathbf{rows}(\alpha) - 2, 0, 0) \end{array} \right. \qquad \mathbf{Draw} := \left| \begin{array}{l} \delta \leftarrow (0.05\ \ 0.10\ \ 0.25\ \ 0.60\ \ 1.0\ \ 2.0)^T \\ \textbf{for}\ \ i \in 0..5 \\ \quad \textbf{for}\ \ j \in 0..8 \\ \quad\quad \alpha_{j+i\cdot 9} \leftarrow \delta_i \\ \alpha \end{array} \right.$$

**Compute interpolated capacity values**

$$\mathbf{Cap} := \frac{\overrightarrow{\mathbf{h}(\mathbf{Draw}, \mathbf{Temp})}}{\%}$$

$\mathbf{Cap}^T =$

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 62.85701 | 74.30008 | 84.28965 | 91.79741 | 96.74373 | ... |

## Regression

### Solution Setup

$\mathbf{N} := \mathbf{rows}(\mathbf{Temp}) \qquad \mathbf{N} = 54$

$\mathbf{i} := 0..\ \mathbf{N} - 1$

$\mathbf{M} := \mathbf{augment}(\mathbf{Temp}, \mathbf{Draw})$

$\mathbf{V} := \mathbf{Cap}$

$\mathbf{n} := 3$

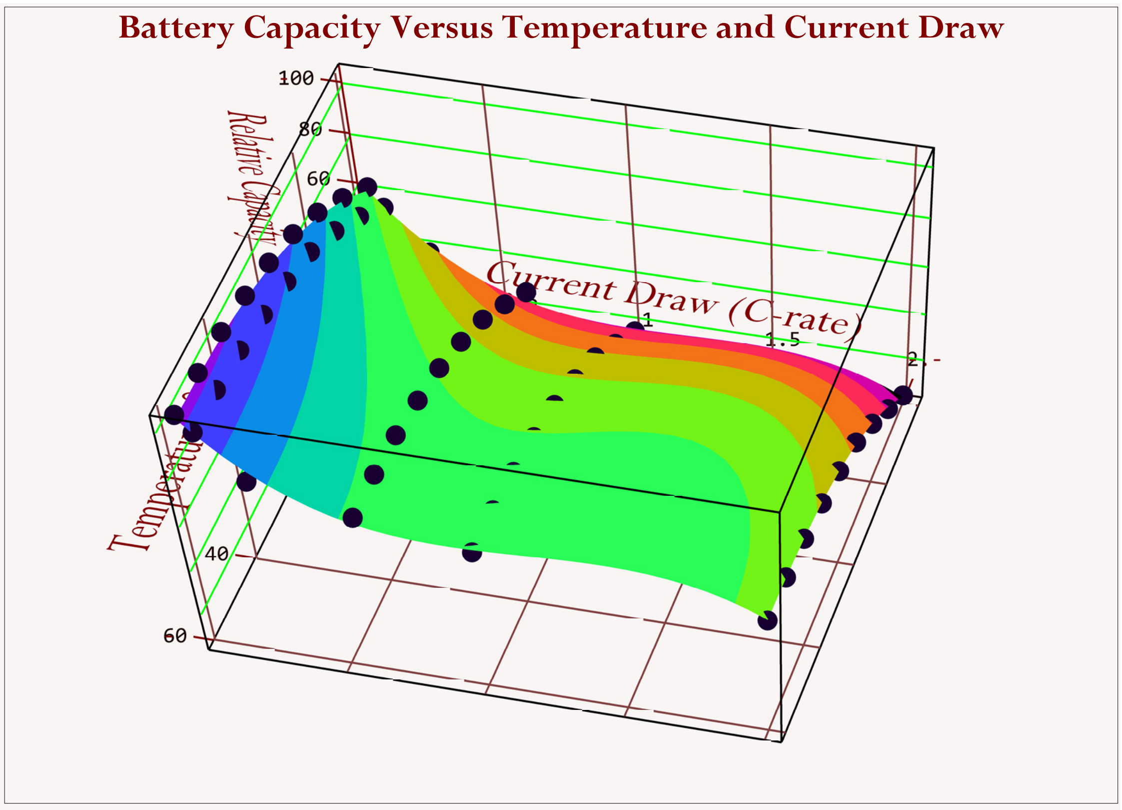
### Solution

$\mathbf{R} := \mathbf{regress}(\mathbf{M}, \mathbf{V}, \mathbf{n})$

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) := \mathbf{interp}\left[\mathbf{R}, \mathbf{M}, \mathbf{V}, \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}\right]$$

### Graphical Display

$\mathbf{F} := \mathbf{CreateMesh}(\mathbf{f}, \mathbf{min}(\mathbf{Temp}), \mathbf{max}(\mathbf{Temp}), \mathbf{min}(\mathbf{Draw}), \mathbf{max}(\mathbf{Draw}), 30, 30)$




**F**, (**Temp**, **Draw**, **Cap**)

### Polynomial Component Extraction

Long Equation Supplied By Mathcad For Coefficient Extraction

$\mathbf{I} := \mathbf{COrder}(\mathbf{Nvars}, \mathbf{deg}) \qquad \mathbf{coeffs} := \mathbf{submatrix}(\mathbf{R}, 3, \mathbf{rows}(\mathbf{R}) - 1, 0, 0)$

$\mathbf{I} =$

| | 0 | 1 |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 0 | 3 |
| 2 | 0 | 2 |
| 3 | 0 | 1 |
| 4 | 1 | 1 |
| 5 | 2 | 1 |
| 6 | 0 | 0 |
| 7 | 1 | 0 |
| 8 | 2 | 0 |
| 9 | 3 | 0 |

$\mathbf{coeffs} =$

| | 0 |
|---|---|
| 0 | -0.0251186354 |
| 1 | -30.2531486509 |
| 2 | 108.2473682202 |
| 3 | -124.9675712504 |
| 4 | 0.1135852987 |
| 5 | -0.0002564402 |
| 6 | 88.4341648373 |
| 7 | 0.8234804414 |
| 8 | -0.0105716421 |
| 9 | 0.0000383771 |

The first column of I gives the power of the x term of each monomial corresponding to the power of the y term given in the second column. The rows of I and **coeffs** correspond. Therefore, you can define the model function using the following summation.

$$\mathbf{poly}(\mathbf{x},\mathbf{y}) := \sum_{\mathbf{i}=0}^{\mathbf{last}(\mathbf{coeffs})} \left(\mathbf{coeffs_i}\cdot \mathbf{x}^{\mathbf{I}_{\mathbf{i},0}}\cdot \mathbf{y}^{\mathbf{I}_{\mathbf{i},1}}\right)$$

Comparing the Mathcad interpolator to the generated polynomial, we get the same results.

$\mathbf{poly}(33.5, 0.05) = 99.79176$

$\mathbf{f}(33.5, 0.05) = 99.79176$

**My explicit polynomial agrees with Mathcad regression formula ⇨ I understand the implementation**

$\mathbf{h}(0.05, 33.5) = 1.0152$

**Here is the value I would ideally have**

## Generate Example Table

**Many customers are not comfortable with graphs and prefer to have a table. Here is the table of capacities computed using the regression formula.**

$\tau := (-20\ \ -10\ \ 0\ \ 10\ \ 20\ \ 30\ \ 40\ \ 50\ \ 60)^{\mathbf{T}}$ $\delta := (0.05\ \ 0.10\ \ 0.25\ \ 0.60\ \ 1.0\ \ 2.0)^{\mathbf{T}}$

```
q := | "Crosscheck"
     | for i ∈ 0.. rows(τ) − 1
     |   for j ∈ 0.. rows(δ) − 1
     |     q_i,j ← f(τ_i, δ_j)
     | q
```

| % Cap | Load Current (C-Rate) | | | | | |
|---|---|---|---|---|---|---|
| Temp (°C) | 0.05 | 0.10 | 0.25 | 0.60 | 1.00 | 2.00 |
| -20 | 61 | 56 | 42 | 24 | 19 | 6 |
| -10 | 73 | 68 | 54 | 36 | 31 | 19 |
| 0 | 82 | 77 | 63 | 46 | 41 | 29 |
| 10 | 90 | 84 | 71 | 54 | 50 | 38 |
| 20 | 95 | 90 | 77 | 60 | 56 | 44 |
| 30 | 99 | 94 | 80 | 64 | 60 | 49 |
| 40 | 101 | 96 | 83 | 66 | 63 | 52 |
| 50 | 102 | 97 | 84 | 68 | 65 | 54 |
| 60 | 102 | 97 | 85 | 69 | 65 | 55 |

$(\tau\ \ \delta^{\mathbf{T}}\ \ \mathbf{q})$

**Here is the same table generated using the interpolation formula (more accurate but requires an Excel macro)**

```
q := | "Crosscheck"
     | for i ∈ 0.. rows(τ) − 1
     |   for j ∈ 0.. rows(δ) − 1
     |     q_i,j ← h(δ_j, τ_i) / %
     | q
```

| % Cap | Load Current (C-Rate) | | | | | |
|---|---|---|---|---|---|---|
| Temp (°C) | 0.05 | 0.10 | 0.25 | 0.60 | 1.00 | 2.00 |
| -20 | 63 | 55 | 39 | 28 | 17 | 6 |
| -10 | 74 | 67 | 50 | 39 | 29 | 18 |
| 0 | 84 | 78 | 59 | 49 | 40 | 29 |
| 10 | 92 | 85 | 67 | 57 | 48 | 39 |
| 20 | 97 | 90 | 72 | 63 | 55 | 44 |
| 30 | 100 | 94 | 76 | 67 | 59 | 49 |
| 40 | 103 | 96 | 79 | 70 | 62 | 52 |
| 50 | 104 | 97 | 80 | 71 | 64 | 54 |
| 60 | 104 | 98 | 80 | 72 | 65 | 55 |

$(\tau\ \ \delta^{\mathbf{T}}\ \ \mathbf{q})$

**Here is the same table generated using the interpolation formula (more accurate)**