Units

$mN := 0.001 \cdot N$ Definition of milliNewton

$AU := 1.49597870700 \cdot 10^{8} \cdot km$ Definition of Astronomical Unit

Constants

$\eta := 61\%$ Efficiency of the ion engine energy conversion

$v := 30 \frac{km}{s}$ Xenon exhaust gas velocity Link

$P := 2.3kW$ This is the maximum power level. The amount of solar power will vary depending on the location of the spacecraft.

$M_{Fuel} := 425kg$ Fuel mass

$M_{BOL} := 1240kg$ Mass at the beginning of life. It will reduce in mass over time as the fuel is expended.

Analysis

Thrust/Power Relationship

Given

$P \cdot \eta = \frac{1}{2} \cdot m' \cdot v^2$ — The total power, minus efficiency losses, is translated into molecule kinetic energy.

$I_{sp} = \frac{v}{g}$ — Specific impulse is a commonly used rocket figure of merit.

$F_{Ion} = m' \cdot v$ — Thrust is equal to the rate of momentum change.

$$\begin{pmatrix} eq1 \\ eq2 \\ eq3 \end{pmatrix} := \text{Find}\left(F_{Ion}, m', v\right) \rightarrow \begin{pmatrix} \frac{2 \cdot P \cdot \eta}{I_{sp} \cdot g} \\ \frac{2 \cdot P \cdot \eta}{I_{sp}^2 \cdot g^2} \\ I_{sp} \cdot g \end{pmatrix}$$

Formula for thrust force

Formula for fuel burn rate

Formula for specific impulse

$$\begin{pmatrix} eq1 \\ eq2 \\ eq3 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{2 \cdot P \cdot \eta}{I_{sp} \cdot g} \\ \frac{2 \cdot P \cdot \eta}{I_{sp}^2 \cdot g^2} \\ I_{sp} \cdot g \end{pmatrix}$$

$I_{sp} := \frac{v}{g} = 3059.1\,s$ — Specified value is 3120 s.

$I_{sp} := 3120\,s$ — I will set the value used here to the specified value.

$F_{Ion}(P, I_{sp}, \eta) := eq1 \rightarrow \frac{2 \cdot P \cdot \eta}{I_{sp} \cdot g}$ — Ion engine thrust.

$m'(P, I_{sp}, \eta) := eq2 \rightarrow \frac{2 \cdot P \cdot \eta}{I_{sp}^2 \cdot g^2}$ — Ion engine fuel consumption.

Dawn Engine Parameters

$F_{Ion} := F_{Ion}(P, I_{sp}, \eta) = 91.70909 \cdot mN$ ✓ Maximum thrust from the engine Link

$m' := m'(P, I_{sp}, \eta) = 0.25897 \cdot \frac{kg}{day}$ ✓ Fuel burn rate at max thrust Link

$T_{Mission} := \frac{M_{Fuel}}{m'} = 39386.6734 \cdot hr$ ✓ Time at full thrust. Solar array power is rated at 1 AU. Power will be reduced at other distances. Link

$\Delta V := \frac{T_{Mission} \cdot F_{Ion}}{M_{BOL}} = 10.48679 \cdot \frac{km}{s}$ ✓ Total mission velocity change Link

Solar Array Power

The two 2.3 x 8.3 meter solar arrays, composed of InGaP/InGaAs/Ge triple-junction cells, provide 1**0.3 kW at 1 AU** (**1.3 kW at end-of-life at 3 AU**) to drive the spacecraft (22-35 V) and the solar electric ion propulsion system (80-140 V).

$$\frac{10.3\,\mathrm{kW}}{\left(\frac{2.77\,\mathrm{AU}}{1\,\mathrm{AU}}\right)^2} = 1342.38684\,\mathrm{W}$$

Link

Ceres is 2.77 AU from the Sun. The rated power at 1AU and the final distance of Dawn from the Sun produce the specification shown.