Derivation

Rise Time

$$\tau_{Rise} := \frac{2V_{CC}}{3} = V_{CC} - V_D + \left(\frac{V_{CC}}{3} - V_{CC} + V_D\right)\cdot e^{-\frac{\tau_{Rise}}{(R_2+R_4)\cdot C_1}} \text{ solve}, \tau_{Rise} \rightarrow$$

$eq3 := \tau_{Rise} \rightarrow$

$T_{Rise}(R_3, R_5, V_D, V_{CC}, C_1) := eq3 \rightarrow$

Fall Time

$$\tau_{Fall} := \frac{V_{CC}}{3} = V_D + \left(\frac{2}{3}\cdot V_{CC} - V_D\right)\cdot e^{-\frac{\tau_{Fall}}{(R_3+R_5)\cdot C_1}} \text{ solve}, \tau_{Fall} \rightarrow$$

$eq4 := \tau_{Fall} \rightarrow$

$T_{Fall}(R_2, R_4, V_D, V_{CC}, C_1) := eq4 \rightarrow$

Period and Frequency

$$T(R_2, R_3, R_{POT}, V_{CC}, V_D, C_1) := \tau_{Rise} + \tau_{Fall} \left| \begin{array}{l} \text{simplify} \\ \text{substitute}, R_4 = k\cdot R_{POT} \\ \text{substitute}, R_5 = (1-k)\cdot R_{POT} \end{array} \right. \rightarrow -C_1\cdot \ln\left(-\frac{V_{CC} - 3\cdot V_D}{3\cdot V_D - 2\cdot V_{CC}}\right)\cdot(R_2 + R_3 + R_{POT})$$

$$f(R_2, R_3, R_{POT}, V_{CC}, V_D, C_1) := \frac{1}{T(R_2, R_3, R_{POT}, V_{CC}, V_D, C_1)}$$

Duty Cycle

$$DC(R_2, R_3, R_{POT}, k, V_{CC}, V_D, C_1) := \frac{T_{Rise}(R_3, R_5, V_D, V_{CC}, C_1)}{T(R_2, R_3, R_{POT}, V_{CC}, V_D, C_1)} \left| \begin{array}{l} \text{simplify} \\ \text{substitute}, R_4 = k\cdot R_{POT} \\ \text{substitute}, R_5 = (1-k)\cdot R_{POT} \end{array} \right. \rightarrow \frac{R_2 + R_{POT}\cdot k}{R_2 + R_3 + R_{POT}}$$

Example

$R_2 := 1k\Omega$ $R_3 := 1k\Omega$ $R_{POT} := 250k\Omega$ $k := 0.5$ $C_1 := 10nF$ $V_D := 0.384V$ $V_{CC} := 5V$

$DC(R_2, R_3, R_{POT}, k, V_{CC}, V_D, C_1) = 0.5$

$f(R_2, R_3, R_{POT}, V_{CC}, V_D, C_1) = 476.58819\cdot Hz$

$$eq3 \rightarrow -C_1 \cdot \ln\left(\frac{V_D - \frac{V_{CC}}{3}}{V_D - \frac{2 \cdot V_{CC}}{3}}\right) \cdot \left(R_2 + R_4\right)$$

$$\tau_{Fall} \rightarrow -C_1 \cdot \ln\left(\frac{V_D - \frac{V_{CC}}{3}}{V_D - \frac{2 \cdot V_{CC}}{3}}\right) \cdot \left(R_3 + R_5\right)$$

$$eq4 \rightarrow -C_1 \cdot \ln\left(\frac{V_D - \frac{V_{CC}}{3}}{V_D - \frac{2 \cdot V_{CC}}{3}}\right) \cdot \left(R_3 + R_5\right)$$